Fermion Mass Hierarchies and Flavor Mixing from T' Symmetry

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Abstract

We construct a supersymmetric model based on $T' \otimes Z_3 \otimes Z_9$ flavor symmetry. At the leading order, the charged lepton mass matrix is not diagonal, T' is broken completely, and the hierarchy in the charged lepton masses is generated naturally. Nearly tri-bimaximal mixing is predicted, subleading effects induce corrections of order λ^2, where λ is the Cabibbo angle. Both the up quark and down quark mass matrices textures of the well-known $U(2)$ flavor theory are produced at the leading order, realistic hierarchies in quark masses and CKM matrix elements are obtained. The vacuum alignment and subleading corrections are discussed in detail.

PACS numbers: 11.30.Hv, 14.60.Pq, 12.15.Ff, 12.60.Jv

I. INTRODUCTION

Experimental data on the quark and lepton masses and mixing provide important clues to the nature of new physics beyond the Standard Model(SM). However, in SM the Yukawa coupling constants which are responsible for the fermion masses and mixing, can be freely adjusted without disturbing the internal consistency of the theory, one must rely on experiments to fix their values. The origin of fermion mass hierarchies and flavor mixing is a longstanding puzzle in the SM of particle physics.

Family symmetry is a fascinating idea to this issue. Current data strongly suggests that there should be a new symmetry that acts horizontally across the three standard model family[1]. Ideally, only the top quark Yukawa coupling is allowed by this symmetry, and all the remaining couplings are generated, as this symmetry is spontaneously broken down. In the original work of Froggatt and Nielsen, they suggested the continuous Abelian $U(1)$ as the flavor symmetry, its spontaneous breaking produces the correct orders of quark mass hierarchies and Cabibbo-Kobayashi-Maskawa (CKM) matrix elements[2]. Models with various horizontal symmetries gauged or global, continuous or discrete, Abelian or non-Abelian, have been proposed[3]. Recently, it is found that discrete group A_4 is especially suitable to derive the so-called tri-bimaximal(TB) mixing[4] in the lepton sector in a natural way[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. The left-handed electroweak lepton doublets $l_i (i = 1, 2, 3)$ transform as A_4 triplet, the right-handed charged leptons e^c, μ^c and τ^c transform as $\mathbf{1}$, $\mathbf{1}''$ and $\mathbf{1}'$ respectively, and two triplets φ_T and φ_S and a singlet ξ are introduced to break the A_4 symmetry spontaneously[11]. If we adopt for quark the same classification scheme under A_4 that we have used for leptons, an identity CKM mixing matrix is obtained at the leading order, which is a good first order approximation. The non leading corrections in the up and down quark sector almost exactly cancel in the mixing matrix. It seems very difficult to implement A_4 as a family symmetry for both the quark and lepton sectors.

Double tetrahedral group T' has three inequivalent irreducible doublet representations $\mathbf{2}$, $\mathbf{2}'$, $\mathbf{2}''$ in addition to the triplet representation $\mathbf{3}$ and three singlet representations $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$ as A_4. Furthermore, the kronecker products of the triplet and singlet representations are identical to those of A_4. Therefore T' can reproduce the success of A_4 model building in the lepton sector, and T' as a family symmetry for both quark and lepton has been

considered[22, 23, 24, 25, 26, 27, 28]. In Ref.[23] a supersymmetric (SUSY) model with $T' \otimes Z_3 \otimes U(1)_{FN}$ flavor symmetry is presented, which is identical to A_4 in the lepton sector. While the quark doublet and the antiquarks of the third generations transforms as $\mathbf{1}$ under T', the other quark doublets and the antiquarks transforms as $\mathbf{2}''$. TB mixing is derived naturally as in A_4 model, whereas only the masses of the second and the third generation quarks and the mixing between them are generated at the leading order. The masses and mixing angles of the first generation quark are induced by higher dimensional operators. The authors built a model with $T' \otimes Z_{12} \otimes Z_{12}$ flavor symmetry in the context of SU(5) grand unification in Ref.[24]. Both the quarks and leptons are assumed to transform as $\mathbf{2} \oplus \mathbf{1}$ under T' in Ref.[26]. A renormalizable model with $T' \otimes Z_2 \otimes Z'_2 \otimes Z''_2$ flavor symmetry is presented in Ref.[28], where the flavor symmetry breaking scale is very low in the range 1 GeV-10 GeV.

The T' symmetry can replicate the success of A_4 model, and it allows the heavy third family to to be treated differently, therefore T' is a very promising flavor symmetry to understand the origin of fermion mass hierarchies and flavor mixing. In this work we shall build a SUSY model based on the $T' \otimes Z_3 \otimes Z_9$ flavor symmetry, the transformation rules of l_i, e^c, μ^c and τ^c are the same as those in the A_4 model[11]. In the quark sector, we exploit the singlet and doublet representation. The fermion mass hierarchies are generated via the spontaneous breaking of the discrete flavor symmetry in contrast with Ref.[11, 23]. The Yukawa matrices of the up and down quarks have the same textures as those in the well-known U(2) flavor theory[29]. The hierarchies in the masses of the known quarks and leptons, the realistic pattern of CKM matrix elements and the TB mixing are naturally produced.

The paper is organized as follows. In section II we present the current experimental data and the parameterizations of fermion mass hierarchies and flavor mixing. A short review of model with U(2) flavor symmetry is given in section III; In section IV a model with $T' \otimes Z_3 \otimes Z_9$ flavor symmetry is constructed, its basic features and predictions are discussed. We present the vacuum alignment and the subleading corrections to the leading order results in section V and section VI respectively. We summarize our results in section VII. Appendix A gives the basic properties of the T' group. The corrections to the vacuum alignment induced by higher dimensional operators are discussed in Appendix B.

II. CURRENT EXPERIMENTAL DATA ON FERMION MASS HIERARCHIES AND FLAVOR MIXING AND THEIR PARAMETERIZATIONS

The observed fermion mass hierarchy is apparent in the quark sector. The masses of up type quarks are[30]

$$\begin{aligned} m_u &\simeq 1.5 - 3\text{MeV} \\ m_c &\simeq 1.16 - 1.34\text{GeV} \\ m_t &\simeq 170.9.1 - 177.5\text{GeV} \end{aligned} \tag{1}$$

and the masses of down type quarks are

$$\begin{aligned} m_d &\simeq 3 - 7\text{MeV} \\ m_s &\simeq 70 - 120\text{MeV} \\ m_b &\simeq 4.13 - 4.27\text{GeV} \end{aligned} \tag{2}$$

We note that all the quark masses except the top quark mass are given in the $\overline{\text{MS}}$ scheme. The light u, d, s quark masses are estimates of so-called current quark mass at the scale about 2 GeV. There is some ambiguity in the measurement of the absolute quark masses since they are scheme dependent, but the ratios of the masses are more concrete

$$\begin{aligned} &\frac{m_u}{m_d} \simeq 0.3 - 0.6 \\ &\frac{m_s}{m_d} \simeq 17 - 22 \\ &\frac{m_s - (m_u + m_d)/2}{m_d - m_u} \simeq 30 - 50 \end{aligned} \tag{3}$$

The masses of the charged leptons have been measured much more unambiguously than the quark masses. The charged lepton sector is also seen to exhibit a large mass hierarchy. Their masses are measured to be[30]

$$\begin{aligned} m_e &\simeq 0.511\text{MeV} \\ m_\mu &\simeq 105.7\text{MeV} \\ m_\tau &\simeq 1777\text{MeV} \end{aligned} \tag{4}$$

The e, μ and τ masses are the pole masses, and their mass hierarchy is similar to that in the down type quark sector. Including the renormalization group equation evolution, the

fermion mass ratios at the GUT scale are parameterized in terms of the Cabibbo angle $\lambda \simeq 0.23$ as follows[31, 32, 33]

$$
\begin{aligned}
&\frac{m_u}{m_t} \sim \lambda^8, \qquad && \frac{m_c}{m_t} \sim \lambda^4, \\
&\frac{m_d}{m_b} \sim \lambda^4, \qquad && \frac{m_s}{m_b} \sim \lambda^2, \\
&\frac{m_e}{m_\tau} \sim \lambda^4, \qquad && \frac{m_\mu}{m_\tau} \sim \lambda^2 \\
&\frac{m_b}{m_t} \sim \lambda^3
\end{aligned}
\tag{5}
$$

Recent precision measurements have greatly improved the knowledge of the CKM matrix, the experimental constraints on the CKM mixing parameters are[30]

$$
|V^{\rm Exp}_{\rm CKM}| \simeq \begin{pmatrix} 0.97377 \pm 0.00027 & 0.2257 \pm 0.0021 & (4.31 \pm 0.30) \times 10^{-3} \\ 0.230 \pm 0.011 & 0.957 \pm 0.095 & (41.6 \pm 0.6) \times 10^{-3} \\ (7.4 \pm 0.8) \times 10^{-3} & (40.6 \pm 2.7) \times 10^{-3} & > 0.78 \; at \; 95\% \; CL \end{pmatrix} \tag{6}
$$

The hierarchy in the quark mixing angles is clearly presented in the Wolfenstein's parameterization of the CKM matrix[30]. Considering the scaling factor associated with the renormalization group evolution of the CKM mixing angles from the electroweak scale to the high scale, the magnitudes of the CKM matrix elements are given in powers of λ as follows

$$
|V_{us}| \sim \lambda\,, \quad |V_{cb}| \sim \lambda^2\,, \quad |V_{td}| \sim \lambda^3, \quad |V_{ub}| \sim \lambda^4 \tag{7}
$$

Observations in the neutrino sector currently provide the strongest indication for physics beyond the standard model. Including the new data released by the MINOS and KamLAND collaborations, the global fit of neutrino oscillation data at 2σ indicates the following values for the lepton mixing angles[34]

$$
0.28 \leq \sin^2\theta_{12} \leq 0.37, \quad 0.38 \leq \sin^2\theta_{23} \leq 0.63, \quad \sin^2\theta_{13} \leq 0.033 \tag{8}
$$

and the best fit values are[34]

$$
\sin^2\theta_{12} = 0.32, \quad \sin^2\theta_{23} = 0.50, \quad \sin^2\theta_{13} = 0.007 \tag{9}
$$

The current data within 1σ is well approximated by the so-called TB mixing[4]

$$
U_{\rm TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{10}
$$

which predicts $\sin^2\theta_{12,{\rm TB}} = \frac{1}{3}$, $\sin^2\theta_{23,{\rm TB}} = \frac{1}{2}$ and $\sin^2\theta_{13,{\rm TB}} = 0$.

III. BRIEF REVIEW ON THE THEORY WITH U(2) FLAVOR SYMMETRY

We shall briefly review the theory with U(2) flavor symmetry in the following, which has been described in detail in the literatures[29]. The three generations of the matter fields are assigned to transform as $\mathbf{2}\oplus\mathbf{1}$, the sfermions of the first two generations are exactly degenerate in the limit of unbroken U(2). In the low energy, this degeneracy is lifted by the small symmetry breaking parameters which determine the light fermion Yukawa couplings, therefore the flavor changing neutral current(FCNC) and CP violating phenomena are sufficiently suppressed so that the corresponding experimental bounds are not violated. Three flavon fields ϕ^a, S^{ab} and $A^{ab}(a,b=1,2)$ are introduced, where ϕ is a U(2) doublet, S and A are symmetric and antisymmetric tensors, and they are U(2) triplet and singlet respectively. The hierarchies in the fermion masses and mixing angles arise from the two step flavor symmetry breaking

$$\mathrm{U}(2)\xrightarrow{\epsilon}\mathrm{U}(1)\xrightarrow{\epsilon'} nothing \tag{11}$$

where both ϵ and ϵ' are small parameters with $\epsilon>\epsilon'$. Both ϕ^a and S^{ab} participate in the first stage of symmetry breaking $\mathrm{U}(2)\xrightarrow{\epsilon}\mathrm{U}(1)$ with $\langle\phi^1\rangle=0$, $\langle S^{11}\rangle=\langle S^{12}\rangle=\langle S^{21}\rangle=0$, $\langle\phi^2\rangle=\mathcal{O}(\epsilon)$ and $\langle S^{22}\rangle=\mathcal{O}(\epsilon)$. The last stage of symmetry breaking is accomplished by A^{ab} with $\langle A^{12}\rangle=-\langle A^{21}\rangle=\mathcal{O}(\epsilon')$. The different mass hierarchies in the up sector and the down sector can be understood by the combination of U(2) flavor symmetry and grand unified symmetries[29], then the Yukawa matrices have the following textures

$$\begin{aligned}
Y_U &= \begin{pmatrix} 0 & \epsilon'\rho & 0 \\ -\epsilon'\rho & \epsilon\rho' & x_u\epsilon \\ 0 & y_u\epsilon & 1 \end{pmatrix}\zeta \\
Y_{D,E} &= \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & (1,\pm3)\epsilon & (x_d,x_e)\epsilon \\ 0 & (y_d,y_e)\epsilon & 1 \end{pmatrix}\varsigma
\end{aligned} \tag{12}$$

where $x_i, y_i=\mathcal{O}(1)$ and $\varsigma\ll\zeta$. The model with U(2) flavor symmetry successfully accounts for the quarks masses, the charged lepton masses and the CKM mixing angles, and the phenomenological constraints from FCNC and CP violation are satisfied. It has been shown that the flavor models based on T' symmetry could reproduce the Yukawa matrices in the U(2) flavor theory[35], However, these models predicted the excluded small mixing angle

solution in the lepton sector. In the following we will use triplet representation in the lepton sector to derive the TB mixing naturally, singlet and doublet representations are exploited in the quark sector, the Yukawa matrices in U(2) model are generated at the leading order. Both the vacuum alignment and the next to leading order corrections are discussed, which are crucial to the flavor model building, however, these issues are omitted in Ref.[35].

IV. THE SUSY MODEL WITH $T' \otimes Z_3 \otimes Z_9$ FLAVOR SYMMETRY

In our scheme, the symmetry group is $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_F$, where G_F is the global flavor symmetry group $G_F = T' \otimes Z_3 \otimes Z_9$. The Z_3 symmetry is to guarantee the correct misalignment in flavor space between the neutrino masses and the charged lepton masses as in Ref.[11, 23], and Z_9 is crucial to obtain the realistic hierarchies in the fermion masses and mixing angles. In addition to the minimal supersymmtric standard model (MSSM) matter fields, we need to introduce the fields which are responsible for the flavor symmetry breaking, we refer to these fields as flavons which are gauge singlets. Both the MSSM fields and the flavon fields and their transformation properties under $T' \otimes Z_3 \otimes Z_9$ are shown in Table I, where α and β are respectively the generators of Z_3 and Z_9 with $\alpha = \exp[i2\pi/3]$ and $\beta = \exp[i2\pi/9]$. Note that although the flavons θ' and χ are not involved in the leading order Yukawa superpotential, they play an important role in the vacuum alignment mechanism.

Fields	ℓ	e^c	μ^c	τ^c	Q_L	U^c	D^c	Q_3	t^c	b^c	$H_{u,d}$	φ_T	φ_S	$\xi, \tilde{\xi}$	ϕ	θ''	θ'	Δ	$\bar{\Delta}$	χ
T'	3	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{2}'$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}'$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_3	α	α^2	α^2	α^2	α	α^2	α^2	α	α^2	α^2	$\mathbf{1}$	$\mathbf{1}$	α	α	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_9	$\mathbf{1}$	$\mathbf{1}$	β^6	β^8	β^3	β^3	β	$\mathbf{1}$	$\mathbf{1}$	β^7	$\mathbf{1}$	β	$\mathbf{1}$	$\mathbf{1}$	β^6	β	β	β^2	β^4	β

TABLE I: The transformation rules of the MSSM fields and the flavon fields under the flavor symmetry $T' \otimes Z_3 \otimes Z_9$. We denote $Q_L = (Q_1, Q_2)^T$, where $Q_1 = (u_L, d_L)^T$ and $Q_2 = (c_L, s_L)^T$ are the electroweak $SU(2)_L$ doublets of the first two generations. $U^c = (u^c, c^c)^T$ and $D^c = (d^c, s^c)^T$, Q_L, U^c and D^c are T' doublets. $Q_3 = (t_L, b_L)^T$ is the electroweak $SU(2)_L$ doublet of the third generation, Q_3, t^c and b^c are T' singlets . The up type and down type Higgs transform as a singlet under the flavor group.

As we shall demonstrate in section V, at the leading order, the scalar components of

the flavon supermultiplets φ_T, φ_S etc. develop vacuum expectation values(VEV) along the following diractions

$$\begin{aligned}
&\langle\varphi_T\rangle=(v_T,0,0),\quad \langle\varphi_S\rangle=(v_S,v_S,v_S),\quad \langle\phi\rangle=(v_1,0),\\
&\langle\xi\rangle=u_\xi,\quad \langle\tilde{\xi}\rangle=0,\quad \langle\theta'\rangle=u'_\theta,\quad \langle\theta''\rangle=u''_\theta,\\
&\langle\Delta\rangle=u_\Delta,\quad \langle\bar{\Delta}\rangle=\bar{u}_\Delta,\quad \langle\chi\rangle=u_\chi
\end{aligned}\tag{13}$$

The electroweak symmetry is broken by the up and down type Higgs with $\langle H_{u,d}\rangle=v_{u,d}$. As we shall see in the following, in order to obtain the realistic pattern of charged fermion masses and mixing angles, these VEVs should be of the orders

$$|\frac{v_T}{\Lambda}|\approx|\frac{v_S}{\Lambda}|\approx|\frac{v_1}{\Lambda}|\sim\lambda^2,\qquad |\frac{u'_\theta}{\Lambda}|\approx|\frac{u''_\theta}{\Lambda}|\approx|\frac{u_\Delta}{\Lambda}|\approx|\frac{\bar{u}_\Delta}{\Lambda}|\sim\lambda^3\tag{14}$$

where Λ is the cut off scale of the theory, these relations imply that the VEVs of the T' triplets and doublet are required to be of order $\lambda^2\Lambda$, while the VEVs of the T' singlets θ, θ', Δ and $\bar{\Delta}$ are of order $\lambda^3\Lambda$. Naturally u_ξ and u_χ should be of the order $\lambda^2\Lambda\sim\lambda^3\Lambda$ as well. The VEVs of required orders in Eq.(14) can be achieved in a finite portion of the parameter space, which will be illustrated in the discussion of the vacuum alignment.

A. The lepton sector

The Yukawa interactions in the lepton sector are controlled by the superpotential

$$w_\ell=w_e+w_\nu\tag{15}$$

where we have separated the contribution to the neutrino masses and the charged lepton masses, both w_e and w_ν are invariant under the gauge group of the standard model and the flavor symmetry $T'\otimes Z_3\otimes Z_9$. The leading order terms of the Yukawa superpotential w_e are

$$\begin{aligned}
w_e=\,&y_e e^c(\ell\varphi_T)\bar{\Delta}^2H_d/\Lambda^3+h_{e1}e^c(\ell\varphi_S)(\varphi_S\varphi_S)H_d/\Lambda^3+h_{e2}e^c(\ell\varphi_s)'(\varphi_S\varphi_S)''H_d/\Lambda^3\\
&+h_{e3}e^c(\ell\varphi_S)''(\varphi_S\varphi_S)'H_d/\Lambda^3+h_{e4}e^c(\ell\varphi_S)\xi^2H_d/\Lambda^3+y_{\mu1}\mu^c(\ell\phi\phi)'H_d/\Lambda^2+y_{\mu2}\mu^c(\ell\varphi_T)'\Delta H_d/\Lambda^2\\
&+h_{\mu1}\mu^c(\ell\varphi_T)'(\varphi_T\varphi_T)H_d/\Lambda^3+h_{\mu2}\mu^c((\ell\varphi_T)_{\mathbf{3}_S}(\varphi_T\varphi_T)_{\mathbf{3}_S})'H_d/\Lambda^3+h_{\mu3}\mu^c((\ell\varphi_T)_{\mathbf{3}_A}(\varphi_T\varphi_T)_{\mathbf{3}_S})'H_d/\Lambda^3\\
&+h_{\mu4}\mu^c(\ell\varphi_T\varphi_T)'\chi H_d/\Lambda^3+h_{\mu5}\mu^c(\ell\varphi_T\varphi_T)\theta'H_d/\Lambda^3+h_{\mu6}\mu^c(\ell\varphi_T\varphi_T)''\theta''H_d/\Lambda^3+h_{\mu7}\mu^c(\ell\varphi_T)'\chi^2H_d/\Lambda^3\\
&+h_{\mu8}\mu^c(\ell\varphi_T)'\theta'\theta''H_d/\Lambda^3+h_{\mu9}\mu^c(\ell\varphi_T)\chi\theta'H_d/\Lambda^3+h_{\mu10}\mu^c(\ell\varphi_T)\theta''\theta''H_d/\Lambda^3+h_{\mu11}\mu^c(\ell\varphi_T)''\chi\theta''H_d/\Lambda^3\\
&+h_{\mu12}\mu^c(\ell\varphi_T)''\theta'\theta'H_d/\Lambda^3+y_\tau\tau^c(\ell\varphi_T)''H_d/\Lambda+...
\end{aligned}\tag{16}$$

where dots stand for additional operators of order $1/\Lambda^3$, whose contributions to the charged lepton masses vanish at the leading order. The coefficients y_e, $h_{ei}(i=1,2,3,4)$, $y_{\mu1}$, $y_{\mu2}$, $h_{\mu i}(i=1{-}12)$ and y_τ are naturally $\mathcal{O}(1)$ coupling constants. After the electroweak symmetry breaking and the flavor symmetry breaking, the charged lepton mass terms from w_e are

$$
\begin{aligned}
w_e &= y_e\frac{\bar{u}^2_\Delta v_T}{\Lambda^3}v_d e^c e+[3(h_{e1}+h_{e2}+h_{e3})\frac{v^3_S}{\Lambda^3}+h_{e4}\frac{u^2_\xi v_S}{\Lambda^3}]v_d e^c(e+\mu+\tau)\\
&+(iy_{\mu1}\frac{v^2_1}{\Lambda^2}+y_{\mu2}\frac{u_\Delta v_T}{\Lambda^2})v_d\mu^c\mu+(h_{\mu5}\frac{2u'_\theta v^2_T}{3\Lambda^3}+h_{\mu9}\frac{u_\chi u'_\theta v_T}{\Lambda^3}+h_{\mu10}\frac{u''^2_\theta v_T}{\Lambda^3})v_d\mu^c e\\
&+[(h_{\mu1}-\frac{2}{9}h_{\mu2}-\frac{1}{3}h_{\mu3})\frac{v^3_T}{\Lambda^3}+h_{\mu4}\frac{2u_\chi v^2_T}{3\Lambda^3}+h_{\mu7}\frac{u^2_\chi v_T}{\Lambda^3}+h_{\mu8}\frac{u'_\theta u''_\theta v_T}{\Lambda^3}]v_d\mu^c\mu\\
&+(h_{\mu6}\frac{2u''_\theta v^2_T}{3\Lambda^3}+h_{\mu11}\frac{u_\chi u''_\theta v_T}{\Lambda^3}+h_{\mu12}\frac{u'^2_\theta v_T}{\Lambda^3})v_d\mu^c\tau+y_\tau\frac{v_T}{\Lambda}v_d\tau^c\tau\\
&\equiv(y_e\frac{\bar{u}^2_\Delta v_T}{\Lambda^3}+y'_e\frac{v^3_S}{\Lambda^3})v_d e^c e+y'_e\frac{v^3_S}{\Lambda^3}v_d e^c\mu+y'_e\frac{v^3_S}{\Lambda^3}v_d e^c\tau+y_{\mu e}\frac{u'_\theta v^2_T}{\Lambda^3}v_d\mu^c e+y_\mu\frac{v^2_1}{\Lambda^2}v_d\mu^c\mu\\
&+y_{\mu\tau}\frac{u''_\theta v^2_T}{\Lambda^3}v_d\mu^c\tau+y_\tau\frac{v_T}{\Lambda}v_d\tau^c\tau
\end{aligned}
\tag{17}
$$

where $y'_e=3(h_{e1}+h_{e2}+h_{e3})+h_{e4}\frac{u^2_\xi}{v^2_S}$, $y_\mu\approx iy_{\mu1}+y_{\mu2}\frac{u_\Delta v_T}{v^2_1}$, $y_{\mu e}=\frac{2}{3}h_{\mu5}+h_{\mu9}\frac{u_\chi}{v_T}+h_{\mu10}\frac{u''^2_\theta}{u'_\theta v_T}$ and $y_{\mu\tau}=\frac{2}{3}h_{\mu6}+h_{\mu11}\frac{u_\chi}{v_T}+h_{\mu12}\frac{u'^2_\theta}{u''_\theta v_T}$. Therefore at the leading order, the charged lepton mass matrix is given by

$$
M^e=\begin{pmatrix} y_e\frac{\bar{u}^2_\Delta v_T}{\Lambda^3}+y'_e\frac{v^3_S}{\Lambda^3} & y'_e\frac{v^3_S}{\Lambda^3} & y'_e\frac{v^3_S}{\Lambda^3}\\ y_{\mu e}\frac{u'_\theta v^2_T}{\Lambda^3} & y_\mu\frac{v^2_1}{\Lambda^2} & y_{\mu\tau}\frac{u''_\theta v^2_T}{\Lambda^3}\\ 0 & 0 & y_\tau\frac{v_T}{\Lambda}\end{pmatrix}v_d
\tag{18}
$$

Note that the charged lepton mass matrix is no longer diagonal at the leading order in contrast with Ref. [11, 23]. Since the charged lepton masses receive contribution from the VEV of φ_S, T' is completely broken already at the leading order. Whereas T' is broken down to Z_3 at the leading order, then it is broken to nothing by the higher dimensional operators in Ref.[11, 23]. The mass matrix M^e is diagonalized by a biunitary transformation $V^{e\dagger}_R M^e V^e_L=diag(m_e,m_\mu,m_\tau)$, therefore $V^{e\dagger}_L M^{e\dagger}M^e V^e_L=diag(m^2_e,m^2_\mu,m^2_\tau)$. The matrix V^e_L approximately is

$$
V^e_L\approx\begin{pmatrix}1 & s^e_{12} & 0\\ -s^{e*}_{12} & 1 & 0\\ 0 & 0 & 1\end{pmatrix}
\tag{19}
$$

where $s^e_{12}=(\frac{y_{\mu e}}{y_\mu}\frac{u'_\theta v^2_T}{v^2_1\Lambda})^*+\frac{|y'_e|^2}{|y_\mu|^2}\frac{|v_S|^6}{|v_1|^4\Lambda^2}$, and the charged lepton masses are approximately given by

$$
m_e\ \approx\ \left|(y_e\frac{\bar{u}^2_\Delta v_T}{\Lambda^3}+y'_e\frac{v^3_S}{\Lambda^3})v_d\right|
$$

$$\begin{aligned} m_\mu &\approx \left|y_\mu \frac{v_1^2}{\Lambda^2} v_d\right| \\ m_\tau &\approx \left|y_\tau \frac{v_T}{\Lambda} v_d\right| \end{aligned} \tag{20}$$

Therefore the mass ratios are estimated

$$\frac{m_e}{m_\tau} \approx \Big|\frac{y_e}{y_\tau}\frac{\bar{u}_\Delta}{\Lambda^2} + \frac{y'_e}{y_\tau}\frac{v_S^3}{v_T\Lambda^2}\Big| \approx \Big|\frac{y'_e}{y_\tau}\frac{v_S^3}{v_T\Lambda^2}\Big|, \qquad \frac{m_\mu}{m_\tau} \approx \Big|\frac{y_\mu}{y_\tau}\frac{v_1^2}{v_T\Lambda}\Big| \tag{21}$$

From Eq.(14) and Eq.(21), we see that the realistic hierarchies among the charged lepton masses $m_\tau : m_\mu : m_e \approx 1 : \lambda^2 : \lambda^4$ are produced naturally. For the neutrino sector, we have

$$w_\nu = (y_\xi \xi + \tilde{y}_\xi \tilde{\xi})(\ell\ell) H_u H_u/\Lambda^2 + y_S(\varphi_S \ell\ell) H_u H_u/\Lambda^2 + ... \tag{22}$$

after the electroweak and flavor symmetry breaking, w_ν gives rise to the following mass terms for the neutrinos

$$w_\nu = y_\xi \frac{u_\xi}{\Lambda}\frac{v_u^2}{\Lambda}(\nu_e^2 + 2\nu_\mu\nu_\tau) + \frac{2}{3} y_S \frac{v_S}{\Lambda}\frac{v_u^2}{\Lambda}(\nu_e^2 + \nu_\mu^2 + \nu_\tau^2 - \nu_e\nu_\mu - \nu_e\nu_\tau - \nu_\mu\nu_\tau) + ... \tag{23}$$

Therefore at the leading order the neutrino mass matrix is

$$M^\nu = \begin{pmatrix} 2y_\xi \frac{u_\xi}{\Lambda} + \frac{4}{3} y_S \frac{v_S}{\Lambda} & -\frac{2}{3} y_S \frac{v_S}{\Lambda} & -\frac{2}{3} y_S \frac{v_S}{\Lambda} \\ -\frac{2}{3} y_S \frac{v_S}{\Lambda} & \frac{4}{3} y_S \frac{v_S}{\Lambda} & 2y_\xi \frac{u_\xi}{\Lambda} - \frac{2}{3} y_S \frac{v_S}{\Lambda} \\ -\frac{2}{3} y_S \frac{v_S}{\Lambda} & 2y_\xi \frac{u_\xi}{\Lambda} - \frac{2}{3} y_S \frac{v_S}{\Lambda} & \frac{4}{3} y_S \frac{v_S}{\Lambda} \end{pmatrix} \frac{v_u^2}{\Lambda} \tag{24}$$

M^ν is diagonalized by a unitary transformation V_L^ν

$$V_L^{\nu T} M^\nu V_L^\nu = diag(2y_\xi \frac{u_\xi}{\Lambda} + 2y_S \frac{v_S}{\Lambda}, 2y_\xi \frac{u_\xi}{\Lambda}, -2y_\xi \frac{u_\xi}{\Lambda} + 2y_S \frac{v_S}{\Lambda}) \frac{v_u^2}{\Lambda} \tag{25}$$

Where the diagonalization matrix V_L^ν is the tri-bimaximal mixing matrix $V_L^\nu = U_{TB}$, therefore the Maki-Nakagawa-Sakata-Pontecorvo(MNSP) mixing matrix, at this order, is

$$V_{\rm MNSP} = V_L^{e\,\dagger} V_L^\nu \approx \begin{pmatrix} \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} s_{12}^e & \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} s_{12}^e & \frac{1}{\sqrt{2}} s_{12}^e \\ -\frac{1}{\sqrt{6}} + \sqrt{\frac{2}{3}} s_{12}^{e*} & \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} s_{12}^{e*} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{26}$$

We see that the MNSP matrix deviates from the TB mixing pattern due to the corrections from the charged lepton sector, in particular, $(V_{\rm MNSP})_{e3}$ is no longer identically zero

$$\begin{aligned} |(V_{\rm MNSP})_{e3}| &\approx \frac{1}{\sqrt{2}}|s_{12}^e| = \frac{1}{\sqrt{2}}\Big|(\frac{y_{\mu e}}{y_\mu}\frac{u'_\theta v_T^2}{v_1^2\Lambda})^* + \frac{|y'_e|^2}{|y_\mu|^2}\frac{|v_S|^6}{|v_1|^4\Lambda^2}\Big| \\ \tan^2\theta_{23} &\approx 1 \\ \tan^2\theta_{12} &\approx \frac{1}{2} - \frac{3}{4}\Big[\frac{y_{\mu e}}{y_\mu}\frac{u'_\theta v_T^2}{v_1^2\Lambda} + (\frac{y_{\mu e}}{y_\mu}\frac{u'_\theta v_T^2}{v_1^2\Lambda})^* + 2\frac{|y'_e|^2}{|y_\mu|^2}\frac{|v_S|^6}{|v_1|^4\Lambda^2}\Big] \end{aligned} \tag{27}$$

From Eq.(14), we learn that s^e_{12} is of order λ^3, therefore at leading order the MNSP matrix is very close to the TB mixing matrix, and the corrections from the charged lepton sector are very small.

B. The quark sector

The Yukawa interactions in the quark sector are

$$w_q = w_u + w_d \tag{28}$$

For the up quark sector, we have

$$\begin{aligned} w_u \;=\; & y_{u1}(\varphi_T Q_L U^c)\Delta H_u/\Lambda^2 + y_{u2}((Q_L U^c)_{\mathbf{3}}(\phi\phi)_{\mathbf{3}})H_u/\Lambda^2 + y_{u3}(Q_L U^c)'\theta''\Delta H_u/\Lambda^2 \\ & + y_{u4}(Q_L\phi)''t^c H_u/\Lambda + y_{u5}Q_3(U^c\phi)'H_u/\Lambda + y_t Q_3 t^c H_u + ... \end{aligned} \tag{29}$$

In the down quark sector, we obtain

$$\begin{aligned} w_d \;=\; & y_{d1}(\varphi_T Q_L D^c)\bar{\Delta}H_d/\Lambda^2 + y_{d2}(Q_L D^c)'\theta''\bar{\Delta}H_d/\Lambda^2 + y_{d3}(Q_L\phi)''b^c\Delta H_d/\Lambda^2 \\ & + y_{d4}Q_3(D^c\phi)'\Delta H_d/\Lambda^2 + y_{b1}Q_3 b^c\Delta H_d/\Lambda + y_{b2}Q_3 b^c(\varphi_T\varphi_T)h_d/\Lambda^2 + y_{b3}Q_3 b^c\chi^2 h_d/\Lambda^2 \\ & + y_{b4}Q_3 b^c\theta'\theta''/\Lambda^2 ... \end{aligned} \tag{30}$$

After electroweak and flavor symmetry breaking, we have the quark mass terms

$$\begin{aligned} w_q \;=\; & y_{u1}\frac{u_\Delta v_T}{\Lambda^2}v_u cc^c + iy_{u2}\frac{v_1^2}{\Lambda^2}v_u cc^c + y_{u3}\frac{u''_\theta u_\Delta}{\Lambda^2}v_u(uc^c - cu^c) + y_{u4}\frac{v_1}{\Lambda}v_u ct^c + y_{u5}\frac{v_1}{\Lambda}v_u tc^c \\ & + y_t v_u tt^c + y_{d1}\frac{\bar{u}_\Delta v_T}{\Lambda^2}v_d ss^c + y_{d2}\frac{u''_\theta\bar{u}_\Delta}{\Lambda^2}v_d(ds^c - sd^c) + y_{d3}\frac{u_\Delta v_1}{\Lambda^2}v_d sb^c + y_{d4}\frac{u_\Delta v_1}{\Lambda^2}v_d bs^c \\ & + y_b\frac{u_\Delta}{\Lambda}v_d bb^c \end{aligned} \tag{31}$$

where $y_b = y_{b1} + y_{b2}\frac{v_T^2}{u_\Delta\Lambda} + y_{b3}\frac{u_\chi^2}{u_\Delta\Lambda} + y_{b4}\frac{u'_\theta u''_\theta}{u_\Delta\Lambda}$, and the resulting quark mass matrices are

$$\begin{aligned} M^u \;=\; & \begin{pmatrix} 0 & -y_{u3}\frac{u''_\theta u_\Delta}{\Lambda^2} & 0 \\ y_{u3}\frac{u''_\theta u_\Delta}{\Lambda^2} & y_{u1}\frac{u_\Delta v_T}{\Lambda^2} + iy_{u2}\frac{v_1^2}{\Lambda^2} & y_{u5}\frac{v_1}{\Lambda} \\ 0 & y_{u4}\frac{v_1}{\Lambda} & y_t \end{pmatrix} v_u \\ M^d \;=\; & \begin{pmatrix} 0 & -y_{d2}\frac{u''_\theta\bar{u}_\Delta}{\Lambda^2} & 0 \\ y_{d2}\frac{u''_\theta\bar{u}_\Delta}{\Lambda^2} & y_{d1}\frac{\bar{u}_\Delta v_T}{\Lambda^2} & y_{d4}\frac{u_\Delta v_1}{\Lambda^2} \\ 0 & y_{d3}\frac{u_\Delta v_1}{\Lambda^2} & y_b\frac{u_\Delta}{\Lambda} \end{pmatrix} v_d \end{aligned} \tag{32}$$

We see that both M^u and M^d have the same textures as those in the U(2) flavor model[29]. From the Appendix A, we see that under the T' generator T, the quark fields transform as $Q_1 \xrightarrow{T} Q_1,\ Q_2(Q_3, u^c, d^c) \xrightarrow{T} \omega^2 Q_2(Q_3, u^c, d^c)$ and $c^c(t^c, s^c, b^c) \xrightarrow{T} \omega c^c(t^c, s^c, b^c)$. Consequently, if the vacuum expectation value of θ'' vanishes, the above mass matrices are the most general ones invariant under the subgroup Z_3'' generated by the generator T. In this work, u''_θ further breaks Z_3 to nothing. Diagonalizing the quark mass matrices in Eq.(32) using the standard perturbation technique[36, 37], we obtain the quark masses as follows

$$\begin{aligned}
m_u &\approx \Big|\frac{y^2_{u3} y_t u''^2_\theta u^2_\Delta}{(iy_{u2}y_t - y_{u4}y_{u5})v_1^2\Lambda^2} v_u\Big| \\
m_c &\approx \Big|(iy_{u2} - \frac{y_{u4}y_{u5}}{y_t})\frac{v_1^2}{\Lambda^2} v_u\Big| \\
m_t &\approx |y_t v_u| \\
m_d &\approx \Big|\frac{y^2_{d2} u''^2_\theta \bar{u}_\Delta}{y_{d1} v_T \Lambda^2} v_d\Big| \\
m_s &\approx \Big|y_{d1}\frac{\bar{u}_\Delta v_T}{\Lambda^2} v_d\Big| \\
m_b &\approx \Big|y_b \frac{u_\Delta}{\Lambda} v_d\Big|
\end{aligned} \tag{33}$$

and the CKM matrix elements are estimated as

$$\begin{aligned}
& V_{ud} \approx V_{cs} \approx V_{tb} \approx 1 \\
& V^*_{us} \approx -V_{cd} \approx \frac{y_{d2}}{y_{d1}}\frac{u''_\theta}{v_T} - \frac{y_{u3}y_t u''_\theta u_\Delta}{(iy_{u2}y_t - y_{u4}y_{u5})v_1^2} \\
& V^*_{cb} \approx -V_{ts} \approx (\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{v_1}{\Lambda} \\
& V^*_{ub} \approx -\frac{y_{u3}y_t}{iy_{u2}y_t - y_{u4}y_{u5}}(\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{u''_\theta u_\Delta}{v_1\Lambda} + \frac{y_{d2}y^*_{d4}}{|y_b|^2}\frac{u''_\theta \bar{u}_\Delta v^*_1}{u_\Delta \Lambda^2} \\
& V_{td} \approx \frac{y_{d2}}{y_{d1}}(\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{u''_\theta v_1}{v_T\Lambda} - \frac{y_{d2}y^*_{d4}}{|y_b|^2}\frac{u''_\theta \bar{u}_\Delta v^*_1}{u_\Delta \Lambda^2}
\end{aligned} \tag{34}$$

From Eq.(14) and Eq.(33), we see that the correct quark mass hierarchies are reproduced $m_t : m_c : m_u \sim 1 : \lambda^4 : \lambda^8$, $m_b : m_s : m_d \sim 1 : \lambda^2 : \lambda^4$ and $m_t : m_b \sim 1 : \lambda^3$. Moreover, Eq.(20) and Eq.(33) imply that the tau lepton and bottom quark masses are respectively of the order λ^2 and λ^3. Since $b-\tau$ unification $m_b \simeq m_\tau$ is usually predicted in many unification models, we expect to achieve $b-\tau$ unification in GUT model with T' flavor symmetry as well, without changing drastically the successful predictions for flavor mixings and fermion mass hierarchies presented here[38]. In our model $\tan\beta \equiv v_u/v_d$ is of order one, the hierarchy between the top quark and bottom quark masses is due to the flavor symmetry breaking

pattern. However, in Ref.[23] the large mass difference between the top and bottom quark is due to large $\tan\beta$, consequently there are large radiative corrections to the quark masses and the CKM matrix elements, which may significantly alter the low energy predictions of quark masses and CKM matrix. From Eq.(14) and Eq.(34), we learn that the correct hierarchy of the CKM matrix elements in Eq.(7) is generated as well. Two interesting relations between the quark masses and mixing angles are predicted

$$\Big|\frac{V_{td}}{V_{ts}}\Big| \approx \sqrt{\frac{m_d}{m_s}}, \qquad \Big|\frac{V_{ub}}{V_{cb}}\Big| \approx \sqrt{\frac{m_u}{m_c}} \tag{35}$$

The above relations are also predicted in U(2) flavor theory. The first relation is satisfied within the large theoretical errors of both sides, and the second relation is not so well fulfilled as the first one. Both relations will be corrected by the next to leading order operators.

V. VACUUM ALIGNMENT

In section IV we have demonstrated that the realistic pattern of fermion masses and flavor mixing are generated, if T' is broken along the directions shown in Eq.(13), in the following we will illustrate that the VEVs in Eq.(13) is really a local minimum of the scalar potential of the model in a finite portion of the parameter space. Using the technique in Ref.[11, 16, 23], a global continuous $U(1)_R$ symmetry is exploited to simplify the vacuum alignment problem, and this symmetry is broken to the discrete R-parity once we include the gaugino masses in the model. The Yukawa superpotentials w_ℓ and w_q in Eq.(15) and Eq.(28) are invariant under the $U(1)_R$ symmetry, if +1 R-charge is assigned to the matter fields (i.e. the lepton and quark superfields), and 0 R-charge to the Higgs and flavon supermultiplets. Since the superpotential must have +2 R-charge, we should introduce some driving fields which carry +2 R-charge in order to avoid the spontaneous breaking of the $U(1)_R$ symmetry, consequently the driving fields enter linearly into the terms of the superpotential. The driving fields and and their transformation properties under $T' \otimes Z_3 \otimes Z_9$ are shown in Table II.

At the leading order, the superpotential depending on the driving fields, which is invariant under all the symmetry of the model, is given by

$$w_v = g_1(\varphi^R_T\phi\phi) + g_2(\varphi^R_T\varphi_T)\Delta + g_3(\phi^R\phi)\chi + g_4(\varphi_T\phi^R\phi) + g_5\chi^R\chi^2 + g_6\chi^R\theta'\theta''$$
$$+g_7\chi^R(\varphi_T\varphi_T) + g_8\theta''^R\theta''^2 + g_9\theta''^R\theta'\chi + g_{10}\theta''^R(\varphi_T\varphi_T)' + M_\Delta\Delta^R\Delta + g_{11}\Delta^R\chi^2$$

Fields	φ_T^R	φ_S^R	ξ^R	ϕ^R	θ''^R	Δ^R	$\bar{\Delta}^R$	χ^R
T'	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2''}$	$\mathbf{1''}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_3	$\mathbf{1}$	α	α	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_9	β^6	$\mathbf{1}$	$\mathbf{1}$	β^2	β^7	β^7	β^5	β^7

TABLE II: The driving fields and their transformation rules under $T' \otimes Z_3 \otimes Z_9$

$$+g_{12}\Delta^R\theta'\theta'' + g_{13}\Delta^R(\varphi_T\varphi_T) + \bar{M}_\Delta\bar{\Delta}^R\bar{\Delta} + g_{14}\bar{\Delta}^R\Delta^2 + g_{15}(\varphi_S^R\varphi_S\varphi_S) + g_{16}(\varphi_S^R\varphi_S)\tilde{\xi}$$
$$+g_{17}\xi^R(\varphi_S\varphi_S) + g_{18}\xi^R\xi^2 + g_{19}\xi^R\xi\tilde{\xi} + g_{20}\xi^R\tilde{\xi}^2 \tag{36}$$

Since there is no distinction between ξ and $\tilde{\xi}$, we define $\tilde{\xi}$ as the field that couples to $(\varphi_S^R\varphi_S)$ in the superpotential w_v as in Ref.[11, 23], and $\tilde{\xi}$ is necessary to achieve the correct vacuum alignment. Similarly the quantum numbers of Δ^R and χ^R are exactly identical, we define Δ^R as the one which couples to Δ.

From the superpotential w_v in Eq.(36), we can derive the scalar potential of this model

$$V = \sum_i |\frac{\partial w_v}{\partial \mathcal{S}_i}|^2 + V_{soft} \tag{37}$$

where $\mathcal{S}_i$ denotes the scalar component of the superfields involved in the model, and V_{soft} includes all possible SUSY soft terms for the scalar fields $\mathcal{S}_i$, and it is invariant under the $T' \otimes Z_3 \otimes Z_9$ flavor symmetry.

$$V_{soft} = \sum_i m^2_{\mathcal{S}_i}|\mathcal{S}_i|^2 + ... \tag{38}$$

where $m^2_{\mathcal{S}_i}$ is the soft mass, and dots stand for other soft SUSY breaking bilinear and trilinear operators. By choosing positive soft mass $m^2_{\mathcal{S}_i}$ for the driving fields, all the driving fields don't acquire VEVs. Since the superpotential w_v is linear in the driving fields, in the SUSY limit all the derivatives with respect to the scalar components of the superfields not charged under $U(1)_R$ symmetry vanish. Therefore in discussing the minimization of the scalar potential, we have to take into account only the derivatives with respect to the scalar components of the driving fields, then we have

$$\begin{aligned}
\frac{\partial w_v}{\partial \varphi^R_{T1}} &= ig_1\phi_1^2 + g_2\varphi_{T1}\Delta = 0 \\
\frac{\partial w_v}{\partial \varphi^R_{T2}} &= (1-i)g_1\phi_1\phi_2 + g_2\varphi_{T3}\Delta = 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial w_v}{\partial \varphi^R_{T3}} &= g_1\phi_2^2 + g_2\varphi_{T2}\Delta = 0\\
\frac{\partial w_v}{\partial \phi^R_{1}} &= g_3\phi_2\chi + g_4'(\varphi_{T1}\phi_2 - (1-i)\varphi_{T3}\phi_1) = 0\\
\frac{\partial w_v}{\partial \phi^R_{2}} &= -g_3\phi_1\chi + g_4'(\varphi_{T1}\phi_1 + (1+i)\varphi_{T2}\phi_2) = 0\\
\frac{\partial w_v}{\partial \chi^R} &= g_5\chi^2 + g_6\theta'\theta'' + g_7(\varphi^2_{T1} + 2\varphi_{T2}\varphi_{T3}) = 0\\
\frac{\partial w_v}{\partial \theta''^R} &= g_8\theta''^2 + g_9\theta'\chi + g_{10}(\varphi^2_{T3} + 2\varphi_{T1}\varphi_{T2}) = 0\\
\frac{\partial w_v}{\partial \Delta^R} &= M_\Delta\Delta + g_{11}\chi^2 + g_{12}\theta'\theta'' + g_{13}(\varphi^2_{T1} + 2\varphi_{T2}\varphi_{T3}) = 0\\
\frac{\partial w_v}{\partial \bar{\Delta}^R} &= \bar{M}_\Delta\bar{\Delta} + g_{14}\Delta^2 = 0\\
\frac{\partial w_v}{\partial \varphi^R_{S1}} &= \frac{2}{3}g_{15}(\varphi^2_{S1} - 2\varphi_{S2}\varphi_{S3}) + g_{16}\varphi_{S1}\tilde{\xi} = 0\\
\frac{\partial w_v}{\partial \varphi^R_{S2}} &= \frac{2}{3}g_{15}(\varphi^2_{S2} - \varphi_{S1}\varphi_{S2}) + g_{16}\varphi_{S3}\tilde{\xi} = 0\\
\frac{\partial w_v}{\partial \varphi^R_{S3}} &= \frac{2}{3}g_{15}(\varphi^2_{S3} - \varphi_{S1}\varphi_{S2}) + g_{16}\varphi_{S2}\tilde{\xi} = 0\\
\frac{\partial w_v}{\partial \xi^R} &= g_{17}(\varphi^2_{S1} + 2\varphi_{S2}\varphi_{S3}) + g_{18}\xi^2 + g_{19}\xi\tilde{\xi} + g_{20}\tilde{\xi}^2 = 0
\end{aligned}
\tag{39}
$$

where $g_4' = \frac{1-i}{2}g_4$, hereafter we simply denote g_4' with g_4 if there is no confusion. These sets of equations admit the solutions

$$
\begin{aligned}
\langle\chi\rangle &= u_\chi\\
\langle\theta'\rangle &= u'_\theta = -\Big[\frac{(g_3^2g_7 + g_4^2g_5)^2g_8}{g_4^4g_6^2g_9}\Big]^{1/3}u_\chi\\
\langle\theta''\rangle &= u''_\theta = \Big[\frac{(g_3^2g_7 + g_4^2g_5)g_9}{g_4^2g_6g_8}\Big]^{1/3}u_\chi\\
\langle\Delta\rangle &= u_\Delta = \frac{g_3^2(g_7g_{12} - g_6g_{13}) + g_4^2(g_5g_{12} - g_6g_{11})}{g_4^2g_6}\frac{u_\chi^2}{M_\Delta}\\
\langle\bar{\Delta}\rangle &= \bar{u}_\Delta = -\frac{[g_3^2(g_7g_{12} - g_6g_{13}) + g_4^2(g_5g_{12} - g_6g_{11})]^2g_{14}}{g_4^4g_6^2}\frac{u_\chi^4}{M_\Delta^2\bar{M}_\Delta}\\
\langle\phi\rangle &= (v_1, 0), \qquad v_1 = \Big(\frac{ig_2g_3[g_3^2(g_7g_{12} - g_6g_{13}) + g_4^2(g_5g_{12} - g_6g_{11})]}{g_1g_4^3g_6}\Big)^{1/2}M_\Delta^{-1/2}u_\chi^{3/2}\\
\langle\varphi_T\rangle &= (v_T, 0, 0), \qquad v_T = \frac{g_3}{g_4}u_\chi\\
\langle\tilde{\xi}\rangle &= 0\\
\langle\xi\rangle &= u_\xi\\
\langle\varphi_S\rangle &= (v_S, v_S, v_S), \qquad v_S = \Big(-\frac{g_{18}}{3g_{17}}\Big)^{1/2}u_\xi
\end{aligned}
\tag{40}
$$

where both u_ξ and u_χ are undetermined, by choosing m_ξ^2 and m_χ^2 to be negative, u_ξ and u_χ would take non-zero values. From Eq.(40), we see that the correct vacuum alignment shown in Eq.(13) is realized. As for the values of the VEVs, we can choose the parameters in the superpotential w_v so that the required orders of the VEVs in Eq.(14) can be achieved.

VI. CORRECTIONS TO THE LEADING ORDER PREDICTIONS FOR THE FERMION MASSES AND FLAVOR MIXING

In the previous section, we have shown that realistic fermion mass hierarchies and flavor mixing are successfully produced at the leading order in our model. However, the leading order results would receive corrections from the higher dimensional operators consistent with the symmetry of the model, which are suppressed by additional powers of Λ. We will study these terms and analyze their physical effects case by case. The next to leading order corrections can be classified into two groups: the first class of corrections are induced by the higher dimensional operators present in the superpotential w_v, which can change the vacuum alignment in Eq.(13), therefore the leading order mass matrices are modified. The second are induced by the higher dimensional operators in the Yukawa superpotentials w_ℓ and w_q, which could modify the Yukawa couplings after the electroweak and flavor symmetry breaking .

A. Higher dimensional operators in the flavon superpotential and the corrections to the vacuum alignment

If we include the next to leading order operators in the flavon superpotential w_v, the vacuum alignment in Eq.(13) would be modified, and the higher order corrections to the vacuum alignment are discussed in detail in the Appendix B. The corrections result in a shift in the VEVs of the scalar fields, and therefore the new vacuum configuration is given by

$$\begin{aligned}&\langle\varphi_T\rangle=(v_T+\delta v_{T1},\delta v_{T2},\delta v_{T3}),\quad\langle\varphi_S\rangle=(v_S+\delta v_{S1},v_S+\delta v_{S2},v_S+\delta v_{S3}),\\&\langle\phi\rangle=(v_1+\delta v_1,\delta v_2),\quad\langle\xi\rangle=u_\xi,\quad\langle\tilde{\xi}\rangle=\delta\tilde{u}_\xi,\quad\langle\theta'\rangle=u'_\theta+\delta u'_\theta,\\&\langle\theta''\rangle=u''_\theta+\delta u''_\theta,\quad\langle\Delta\rangle=u_\Delta+\delta u_\Delta,\quad\langle\bar{\Delta}\rangle=\bar{u}_\Delta+\delta\bar{u}_\Delta,\quad\langle\chi\rangle=u_\chi\end{aligned}\qquad(41)$$

In the Appendix B, we show that the corrections δv_{T2}, δv_{T3}, δv_1, δv_2, $\delta u'_\theta$, $\delta u''_\theta$ and $\delta\bar{u}_\Delta$ arise at order $1/\Lambda$. δv_{T1} and δu_Δ are of order $1/\Lambda^2$, and the corrections δv_{S1}, δv_{S2}, δv_{S3} and $\delta\tilde{u}_\xi$ are suppressed by $1/\Lambda^3$, which are small enough and can be negligible. Note that there should also be corrections to the VEVs of ξ and χ, but we do not have to indicate this explicitly by the addition of terms δu_ξ and δu_χ, since both u_ξ and u_χ are undetermined at the leading order.

Repeating the calculations in section IV and substituting the modified vacuum into the Yukawa superpotentials w_ℓ and w_q, we can obtain the new vacuum corrections to the fermion mass matrices as follows

$$\delta M^e_1 = \begin{pmatrix} y_e\frac{\bar{u}^2_\Delta\delta v_{T1}+2\bar{u}_\Delta\delta\bar{u}_\Delta v_T}{\Lambda^3} & y_e\frac{\bar{u}^2_\Delta\delta v_{T3}}{\Lambda^3} & y_e\frac{\bar{u}^2_\Delta\delta v_{T2}}{\Lambda^3} \\ y_{\mu2}\frac{u_\Delta\delta v_{T2}}{\Lambda^2} & 2iy_{\mu1}\frac{v_1\delta v_1}{\Lambda^2} & (1-i)y_{\mu1}\frac{v_1\delta v_2}{\Lambda^2}+y_{\mu2}\frac{u_\Delta\delta v_{T3}}{\Lambda^2} \\ y_\tau\frac{\delta v_{T3}}{\Lambda} & y_\tau\frac{\delta v_{T2}}{\Lambda} & y_\tau\frac{\delta v_{T1}}{\Lambda} \end{pmatrix} v_d \tag{42}$$

$$\delta M^\nu_1 = \begin{pmatrix} \frac{4}{3}y_S\frac{\delta v_{S1}}{\Lambda} & -\frac{2}{3}y_S\frac{\delta v_{S3}}{\Lambda} & -\frac{2}{3}y_S\frac{\delta v_{S2}}{\Lambda} \\ -\frac{2}{3}y_S\frac{\delta v_{S3}}{\Lambda} & \frac{4}{3}y_S\frac{\delta v_{S2}}{\Lambda} & -\frac{2}{3}y_S\frac{\delta v_{S1}}{\Lambda} \\ -\frac{2}{3}y_S\frac{\delta v_{S2}}{\Lambda} & -\frac{2}{3}y_S\frac{\delta v_{S1}}{\Lambda} & \frac{4}{3}y_S\frac{\delta v_{S3}}{\Lambda} \end{pmatrix} \frac{v_u^2}{\Lambda} \tag{43}$$

$$\delta M^u_1 = \begin{pmatrix} iy_{u1}\frac{u_\Delta\delta v_{T2}}{\Lambda^2} & \delta u_1 & -y_{u5}\frac{\delta v_2}{\Lambda} \\ \delta u'_1 & y_{u1}\frac{u_\Delta\delta v_{T1}+\delta u_\Delta v_T}{\Lambda^2}+2iy_{u2}\frac{v_1\delta v_1}{\Lambda^2} & y_{u5}\frac{\delta v_1}{\Lambda} \\ -y_{u4}\frac{\delta v_2}{\Lambda} & y_{u4}\frac{\delta v_1}{\Lambda} & 0 \end{pmatrix} v_u \tag{44}$$

$$\delta M^d_1 = \begin{pmatrix} iy_{d1}\frac{\bar{u}_\Delta\delta v_{T2}}{\Lambda^2} & \delta d_1 & -y_{d4}\frac{u_\Delta\delta v_2}{\Lambda^2} \\ \delta d'_1 & y_{d1}\frac{\bar{u}_\Delta\delta v_{T1}+\delta\bar{u}_\Delta v_T}{\Lambda^2} & y_{d4}\frac{u_\Delta\delta v_1+\delta u_\Delta v_1}{\Lambda^2} \\ -y_{d3}\frac{u_\Delta\delta v_2}{\Lambda^2} & y_{d3}\frac{u_\Delta\delta v_1+\delta u_\Delta v_1}{\Lambda^2} & y_b\frac{\delta u_\Delta}{\Lambda} \end{pmatrix} v_d \tag{45}$$

where δe_1, $\delta e'_1$, δu_1, $\delta u'_1$, δd_1 and $\delta d'_1$ are given by

$$\begin{aligned} \delta u_1 &= \frac{1-i}{2}y_{u1}\frac{u_\Delta\delta v_{T3}}{\Lambda^2} - iy_{u2}\frac{v_1\delta v_2}{\Lambda^2} - y_{u3}\frac{\delta u''_\theta u_\Delta + u''_\theta\delta u_\Delta}{\Lambda^2} \\ \delta u'_1 &= \frac{1-i}{2}y_{u1}\frac{u_\Delta\delta v_{T3}}{\Lambda^2} - iy_{u2}\frac{v_1\delta v_2}{\Lambda^2} + y_{u3}\frac{\delta u''_\theta u_\Delta + u''_\theta\delta u_\Delta}{\Lambda^2} \\ \delta d_1 &= \frac{1-i}{2}y_{d1}\frac{\bar{u}_\Delta\delta v_{T3}}{\Lambda^2} - y_{d2}\frac{\delta u''_\theta\bar{u}_\Delta + u''_\theta\delta\bar{u}_\Delta}{\Lambda^2} \\ \delta d'_1 &= \frac{1-i}{2}y_{d1}\frac{\bar{u}_\Delta\delta v_{T3}}{\Lambda^2} + y_{d2}\frac{\delta u''_\theta\bar{u}_\Delta + u''_\theta\delta\bar{u}_\Delta}{\Lambda^2} \end{aligned}$$

As for δM^e_1, we have neglected the corrections induced by $\delta v_{Si}(i=1,2,3)$ and $\delta\tilde{u}_\xi$, since they are of higher order $1/\Lambda^3$ and can be negligible comparing with the corrections proportional to $\delta v_{Ti}(i=1,2,3)$ and $\delta\bar{u}_\Delta$. Eq.(43) implies that the corrections to the neutrino mass

matrix are suppressed by additional power of $1/\Lambda^3$ relative to the leading order results. Concerning the quark sector, the correction terms $y_{u3}\frac{\delta u''_{\theta}u_{\Delta}+u''_{\theta}\delta u_{\Delta}}{\Lambda^2}$, $y_{d2}\frac{\delta u''_{\theta}\bar{u}_{\Delta}+u''_{\theta}\delta\bar{u}_{\Delta}}{\Lambda^2}$ and $y_b\frac{\delta u_{\Delta}}{\Lambda}$ can be absorbed by the redefinition of y_{u3}, y_{d2} and y_b respectively.

B. Corrections induced by higher dimensional operators in the Yukawa superpotential

1. Corrections to w_ℓ

The leading order operators relevant to e^c are of order $1/\Lambda^3$, which are shown in Eq.(16), at the next order $1/\Lambda^4$ there are two operators

$$e^c(\ell\varphi_T)\Delta^2\bar{\Delta}H_d/\Lambda^4, \qquad e^c(\ell\phi\phi)\Delta\bar{\Delta}H_d/\Lambda^4 \tag{46}$$

Because $(\phi\phi)_{\mathbf{3}} = (i\phi_1^2, \phi_2^2, (1-i)\phi_1\phi_2)$, its VEV is parallel to that of φ_T. Therefore both operators have the same structure as the leading operator $e^c(\ell\varphi_T)\bar{\Delta}^2H_d/\Lambda^3$, their effects can be absorbed by the redefinition of y_e. Concerning the μ^c relevant terms in the leading order Yukawa superpotential in Eq.(16), they comprise both terms of order $1/\Lambda^2$ and terms of order $1/\Lambda^3$. The subleading operators invariant under the symmetry of the model arise at order $1/\Lambda^5$, and their contributions are completely negligible relative to the corrections from the modified vacuum configuration. The leading operator of the τ^c relevant term is of order $1/\Lambda$, and the next to leading order corrections are of order $1/\Lambda^4$, therefore their contributions can be neglected comparing with the corrections from the new vacuum.

The same arguments used for the charged lepton mass are applicable to the neutrino sector as well. The leading operators contributing to M^ν are of order $1/\Lambda^2$ from Eq.(22), and the leading order results receive corrections from higher dimensional operators of order $1/\Lambda^5$ at the next to leading order. Therefore, the charged lepton mass matrix mainly receives corrections from the modified vacuum configuration. Whereas, the corrections to the neutrino mass matrix from the next to leading order operators in both w_v and w_ν are negligible, and T' is approximately broken to Z_4 subgroup in the neutrino sector, even if higher order corrections are included.

2. Corrections to w_u

As is shown in Eq.(29), the leading operators, which give rise to the M^u_{11}, M^u_{12}, M^u_{21} and M^u_{22}, are of order $1/\Lambda^2$. At the next order $1/\Lambda^3$, there are two operators whose contributions can not be absorbed by parameter redefinition

$$x_{u1}((Q_L U^c)_{\mathbf{3}}(\varphi_T\varphi_T)_{\mathbf{3}_S})'\theta'' H_u/\Lambda^3, \quad x_{u2}((Q_L U^c)_{\mathbf{3}}(\varphi_T\varphi_T)_{\mathbf{3}_S})''\theta' H_u/\Lambda^3 \tag{47}$$

The leading operators contributing to M^u_{13}, M^u_{23}, M^u_{31} and M^u_{32} are of order $1/\Lambda$ from Eq.(29), and the next to leading order corrections arise at order $1/\Lambda^4$. These contributions are negligible relative to the corrections induced by the modified vacuum, which is shown in Eq.(44). The corrections to M^u_{33} can be absorbed by redefining the parameter y_t. As a result, the higher dimensional operators corrections to the up quark mass matrix are

$$\delta M^u_2 = \begin{pmatrix} \frac{2i}{3}x_{u2}\frac{u'_\theta v^2_T}{\Lambda^3} & \frac{1-i}{3}x_{u1}\frac{u''_\theta v^2_T}{\Lambda^3} & 0 \\ \frac{1-i}{3}x_{u1}\frac{u''_\theta v^2_T}{\Lambda^3} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} v_u \tag{48}$$

3. Corrections to w_d

Concerning the M^d_{11}, M^d_{12},M^d_{21} and M^d_{22} relevant operators, the leading terms are of order $1/\Lambda^2$, which are shown in Eq.(30), and there are three operators at the order $1/\Lambda^3$

$$(\varphi_T Q_L D^c)\Delta^2 H_d/\Lambda^3, \quad ((Q_L D^c)_{\mathbf{3}}(\phi\phi)_{\mathbf{3}})\Delta H_d/\Lambda^3, \quad (Q_L D^c)'\theta''\Delta^2 H_d/\Lambda^3 \tag{49}$$

After electroweak and flavor symmetry breaking, the above operators have the same structures as the leading ones, and their contributions can be absorbed by redefinition of y_{d1} and y_{d2}. Nontrivial higher dimensional operators arise at the order $1/\Lambda^4$, their contributions are negligible comparing with the corrections from the modified vacuum, which is shown in Eq.(45). Similarly the next to leading operators contributing to M^d_{13}, M^d_{23}, M^d_{31} and M^d_{32} are of order $1/\Lambda^3$, and only two operators remain after symmetry breaking and parameter redefinition

$$x_{d1}(\varphi_T Q_L\phi)b^c\theta'' H_d/\Lambda^3, \quad x_{d2}Q_3(\varphi_T D^c\phi)''\theta'' H_d/\Lambda^3 \tag{50}$$

Therefore the corrections to the down quark mass matrix from the higher dimensional

operators are

$$\delta M_2^d = \begin{pmatrix} 0 & 0 & ix_{d2}\frac{u''_\theta v_1 v_T}{\Lambda^3} \\ 0 & 0 & 0 \\ ix_{d1}\frac{u''_\theta v_1 v_T}{\Lambda^3} & 0 & 0 \end{pmatrix} v_d \tag{51}$$

C. Fermion masses and flavor mixing including the next to leading order corrections

1. Lepton masses and MNSP matrix

Combining the leading order predictions Eq.(18) for the charged lepton mass matrix with the subleading corrections in Eq.(42), we obtain that the charged lepton mass matrix is modified as

$$\mathcal{M}^e = M^e + \delta M_1^e = \begin{pmatrix} y_e\frac{\bar{u}_\Delta^2 v_T}{\Lambda^3} + y'_e\frac{v_S^3}{\Lambda^3} & y_e\frac{\bar{u}_\Delta^2 \delta v_{T3}}{\Lambda^3} + y'_e\frac{v_S^3}{\Lambda^3} & y_e\frac{\bar{u}_\Delta^2 \delta v_{T2}}{\Lambda^3} + y'_e\frac{v_S^3}{\Lambda^3} \\ \delta e'_2 & y_\mu\frac{v_1^2}{\Lambda^2} & \delta e_2 \\ y_\tau\frac{\delta v_{T3}}{\Lambda} & y_\tau\frac{\delta v_{T2}}{\Lambda} & y_\tau\frac{v_T}{\Lambda} \end{pmatrix} \tag{52}$$

where

$$\begin{aligned} \delta e_2 &= (1-i)y_{\mu1}\frac{v_1\delta v_2}{\Lambda^2} + y_{\mu2}\frac{u_\Delta \delta v_{T3}}{\Lambda^2} + y_{\mu\tau}\frac{u''_\theta v_T^2}{\Lambda^3} \\ \delta e'_2 &= y_{\mu2}\frac{u_\Delta \delta v_{T2}}{\Lambda^2} + y_{\mu e}\frac{u'_\theta v_T^2}{\Lambda^3} \end{aligned}$$

we have set $v_T + \delta v_{T1} \rightarrow v_T$, $v_1 + \delta v_1 \rightarrow v_1$ and $\bar{u}_\Delta + \delta\bar{u}_\Delta \rightarrow \bar{u}_\Delta$. In the neutrino sector, since the corrections from the new vacuum and the higher dimensional operators in the the Yukawa superpotential w_ν are of order $1/\Lambda^5$, as are shown in the previous subsections, these contributions are negligible. Therefore the neutrino mass matrix is approximately not affected by the subleading operators. Performing the same procedure as that in section IV, we see that both the charged lepton masses and the neutrino masses approximately are not modified by the next to leading order operators, and the MNSP matrix becomes

$$V_{\rm MNSP} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}}\frac{\delta v^*_{T3}}{v^*_T} & \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\frac{\delta v^*_{T3}}{v^*_T} & -\frac{1}{\sqrt{2}}\frac{\delta v^*_{T3}}{v^*_T} \\ -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}}\frac{\delta v^*_{T2}}{v^*_T} & \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\frac{\delta v^*_{T2}}{v^*_T} & -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{\delta v^*_{T2}}{v^*_T} \\ -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}}\frac{2\delta v_{T3} - \delta v_{T2}}{v_T} & \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\frac{\delta v_{T2} + \delta v_{T3}}{v_T} & \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{\delta v_{T2}}{v_T} \end{pmatrix} \tag{53}$$

therefore

$$
\begin{aligned}
&|(V_{\rm MNSP})_{e3}| \approx \frac{1}{\sqrt{2}}\Big|\frac{\delta v_{T3}}{v_T}\Big| \\
&\tan^2\theta_{23} \approx 1 + 2\Big(\frac{\delta v_{T2}}{v_T} + \frac{\delta v^*_{T2}}{v^*_T}\Big) \\
&\tan^2\theta_{12} \approx \frac{1}{2} - \frac{3}{4}\Big(\frac{\delta v_{T3}}{v_T} + \frac{\delta v^*_{T3}}{v^*_T}\Big)
\end{aligned}
\tag{54}
$$

We see $\delta v_{T2}/v_T \sim \lambda^2$ and $\delta v_{T3}/v_T \sim \lambda^2$ from the Appendix B, therefore the deviations of the mixing angles from the TB mixing predictions are of order λ^2, which are allowed by the current neutrino oscillation data in Eq.(8).

2. Quark masses and CKM matrix

Including the corrections δM^u_i and $\delta M^d_i (i = 1, 2)$ induced by the new vacuum and the higher dimensional operators in the Yukawa superpotential w_q, the up quark and down quark mass matrices becomes

$$
\mathcal{M}^u \;=\; M^u + \delta M^u_1 + \delta M^u_2 = \begin{pmatrix} iy_{u1}\frac{u_\Delta \delta v_{T2}}{\Lambda^2} + \frac{2i}{3}x_{u2}\frac{u'_\theta v^2_T}{\Lambda^3} & -y_{u3}\frac{u''_\theta u_\Delta}{\Lambda^2} + \delta u_2 & -y_{u5}\frac{\delta v_2}{\Lambda} \\ y_{u3}\frac{u''_\theta u_\Delta}{\Lambda^2} + \delta u_2 & y_{u1}\frac{u_\Delta v_T}{\Lambda^2} + iy_{u2}\frac{v^2_1}{\Lambda^2} & y_{u5}\frac{v_1}{\Lambda} \\ -y_{u4}\frac{\delta v_2}{\Lambda} & y_{u4}\frac{v_1}{\Lambda} & y_t \end{pmatrix} v_u
$$

$$
\mathcal{M}^d \;=\; \begin{pmatrix} iy_{d1}\frac{\bar{u}_\Delta \delta v_{T2}}{\Lambda^2} & -y_{d2}\frac{u''_\theta \bar{u}_\Delta}{\Lambda^2} + \frac{1-i}{2}y_{d1}\frac{\bar{u}_\Delta \delta v_{T3}}{\Lambda^2} & -y_{d4}\frac{u_\Delta \delta v_2}{\Lambda^2} + ix_{d2}\frac{u''_\theta v_1 v_T}{\Lambda^3} \\ y_{d2}\frac{u''_\theta \bar{u}_\Delta}{\Lambda^2} + \frac{1-i}{2}y_{d1}\frac{\bar{u}_\Delta \delta v_{T3}}{\Lambda^2} & y_{d1}\frac{\bar{u}_\Delta v_T}{\Lambda^2} & y_{d4}\frac{u_\Delta v_1}{\Lambda^2} \\ -y_{d3}\frac{u_\Delta \delta v_2}{\Lambda^2} + ix_{d1}\frac{u''_\theta v_1 v_T}{\Lambda^3} & y_{d3}\frac{u_\Delta v_1}{\Lambda^2} & y_b\frac{u_\Delta}{\Lambda} \end{pmatrix} v_d
$$

where $\delta u_2 = \frac{1-i}{2}y_{u1}\frac{u_\Delta \delta v_{T3}}{\Lambda^2} - iy_{u2}\frac{v_1 \delta v_2}{\Lambda^2} + \frac{1-i}{3}x_{u1}\frac{u''_\theta v^2_T}{\Lambda^3}$, and we have set $v_T + \delta v_{T1} \rightarrow v_T$, $v_1 + \delta v_1 \rightarrow v_1$, $u_\Delta + \delta u_\Delta \rightarrow u_\Delta$ and $\bar{u}_\Delta + \delta\bar{u}_\Delta \rightarrow \bar{u}_\Delta$. Diagonalizing the above mass matrices perturbatively, we obtain the quark masses as follows

$$
\begin{aligned}
m_u \;&\approx\; \Big|\Big(iy_{u1}\frac{u_\Delta \delta v_{T2}}{\Lambda^2} - iy_{u2}\frac{\delta v^2_2}{\Lambda^2} + \frac{y^2_{u3} y_t u''^2_\theta u^2_\Delta}{(iy_{u2}y_t - y_{u4}y_{u5})v^2_1\Lambda^2} + \frac{2i}{3}x_{u2}\frac{u'_\theta v^2_T}{\Lambda^3}\Big)v_u\Big| \\
m_c \;&\approx\; \Big|\Big(iy_{u2} - \frac{y_{u4}y_{u5}}{y_t}\Big)\frac{v^2_1}{\Lambda^2}v_u\Big| \\
m_t \;&\approx\; \Big|y_t v_u\Big| \\
m_d \;&\approx\; \Big|\Big(iy_{d1}\frac{\bar{u}_\Delta \delta v_{T2}}{\Lambda^2} + \frac{y^2_{d2}}{y_{d1}}\frac{u''^2_\theta \bar{u}_\Delta}{v_T\Lambda^2}\Big)v_d\Big| \\
m_s \;&\approx\; \Big|y_{d1}\frac{\bar{u}_\Delta v_T}{\Lambda^2}v_d\Big| \\
m_b \;&\approx\; \Big|y_b\frac{u_\Delta}{\Lambda}v_d\Big|
\end{aligned}
\tag{55}
$$

and the CKM matrix elements are approximately given by

$$
\begin{aligned}
V_{ud} &\approx V_{cs} \approx V_{tb} \approx 1 \\
V^*_{us} &\approx -V_{cd} \approx \frac{y_{d2}}{y_{d1}}\frac{u''_\theta}{v_T} + \frac{1-i}{2}\frac{\delta v_{T3}}{v_T} + \frac{\delta v_2}{v_1} - \frac{y_{u3}y_t u''_\theta u_\Delta}{(iy_{u2}y_t - y_{u4}y_{u5})v_1^2} \\
V^*_{cb} &\approx -V_{ts} \approx (\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{v_1}{\Lambda} \\
V^*_{ub} &\approx -\frac{y_{u3}y_t}{iy_{u2}y_t - y_{u4}y_{u5}}(\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{u''_\theta u_\Delta}{v_1\Lambda} + i\frac{x_{d1}}{y_b}\frac{u''_\theta v_1 v_T}{u_\Delta \Lambda^2} \\
V_{td} &\approx \frac{y_{d2}}{y_{d1}}(\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{u''_\theta v_1}{v_T\Lambda} + (\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})(\frac{\delta v_2}{\Lambda} + \frac{1-i}{2}\frac{v_1 \delta v_{T3}}{v_T \Lambda}) - i\frac{x_{d1}}{y_b}\frac{u''_\theta v_1 v_T}{u_\Delta\Lambda^2}
\end{aligned}
\tag{56}
$$

Because $\delta v_{T2}/\Lambda$, $\delta v_{T3}/\Lambda$ and $\delta v_2/\Lambda$ are of order λ^4 from the Appendix B, to get the appropriate magnitude of the up quark mass, we assume that the couplings y_{u1} and x_{u2} are smaller than one by a factor of λ, i.e. $y_{u1} \sim x_{u2} \sim \lambda$. From Eq.(14), Eq.(55) and Eq.(56), we see that the realistic hierarchies in quark masses and CKM matrix elements are generated, and the relations between quark masses and mixing angles in Eq.(35) are no longer satisfied after including the subleading contributions. It is very likely that the higher order contributions would improve the agreement between the model predictions and the experimental data.

VII. SUMMARY AND DISCUSSION

T' is a promising discrete group for a unified description of both quark and lepton mass hierarchies and flavor mixing. T' can reproduce the success of A_4 model building, and T' has advantage over A_4 in extension to the quark sector because it has doublet representations in addition to singlet representations and triplet representation. We have built a SUSY model based on $T' \otimes Z_3 \otimes Z_9$ flavor symmetry, where the fermion mass hierarchies arise from the flavor symmetry breaking which is crucial in producing the flavor mixing as well.

In the lepton sector, the left handed electroweak lepton doublets $l_i (i = 1, 2, 3)$ are T' triplet, and the right handed charged leptons e^c, μ^c and τ^c transform as $\mathbf{1}$, $\mathbf{1}''$ and $\mathbf{1}'$ respectively. The charged lepton mass matrix is no longer diagonal at the leading order, and T' is broken completely in the charged lepton sector. However, it is broken down to the Z_3 subgroup generated by the element T in A_4 model[11] and in the T' model of Ref.[23] at the leading order, then it is further broken to nothing by the subleading operators. The MNSP matrix is predicted to be nearly TB mixing matrix at the leading order, and the deviations

due to the contributions of the charged lepton sector are of order λ^3 and are negligible. In the neutrino sector, T' is broken down to the Z_4 subgroup generated by the element TST^2 at the leading order as Ref.[23]. The higher order corrections to the neutrino mass matrix are strongly suppressed, consequently the Z_4 symmetry almost remains. Considering the next to leading order operators in the Yukawa superpotential w_ℓ and the flavon superpotential w_v, then the mixing angles are predicted to deviate from the TB mixing predictions by terms of order λ^2, which are in the interval indicated by the experimental data at the 3σ level.

In the quark sector, doublet representation are exploited. The first two generations transform as doublet ($\mathbf{2}$ or $\mathbf{2}'$), and the third generation is T' singlet ($\mathbf{1}'$ or $\mathbf{1}''$). At the leading order, both the up and down quark Yukawa matrices textures in U(2) flavor theory are produced, and the correct hierarchies in quark masses and mixing angles are obtained. T' is completely broken at the leading order, this is in contrast with Ref.[23], where the T' flavor symmetry is broken down to Z_3 at the leading order and is further broken to nothing by the next to leading order contributions. After including the corrections induced by the next to leading order operators, the correct orders of quark masses and CKM matrix elements at the leading order remain except the up quark mass, we need to mildly fine-tune the coupling coefficients y_{u1} and x_{u2} to be smaller than one by a factor of λ.

The vacuum alignment and the higher order corrections are discussed in details. We have shown that the scalar potential in the model presents the correct T' breaking alignment in a finite portion of the parameter space, which plays an important role in producing the realistic fermion mass hierarchies and flavor mixing. The VEVs should be of the orders shown in Eq.(14), the minor hierarchy in the VEVs can be achieved by moderately fine-tuning the parameters in w_v. The origin of this hierarchies may be qualitatively understood in the grand unification models[38], in which $b-\tau$ unification may be predicted as well. The higher order corrections are due to the higher dimensional operators which modify the Yukawa couplings and the the vacuum alignment, and they don't spoil the leading order predictions.

Our model is different from the model in Ref.[23] mainly in the following three aspects:

1. We have introduced the auxiliary discrete symmetry Z_9 instead of continuous $U(1)_{FN}$, both the fermion mass hierarchies and flavor mixing arise from the spontaneous breaking of the flavor symmetry, whereas a continuous abelian flavor symmetry $U(1)_{FN}$ is introduced to describe the fermion mass hierarchies in Ref.[23].

2. There are large differences between the two models in the quark sector, at the leading order, the favorable Yukawa matrix textures of the U(2) flavor theory are obtained, and the realistic quark mass hierarchies and CKM matrix elements are produced in our model. However, in the model in Ref.[23], only the masses of the second and third generation quarks and the mixing between them are generated at the leading order, the masses and mixing angles of the first generation quarks are produced via subleading effects induced by the higher dimensional operators.

3. The large mass hierarchy between the top quark and the bottom quark arises from the flavor symmetry breaking, and $\tan\beta \equiv v_u/v_d$ is of order one in our model. Nevertheless, this hierarchy is due to large $\tan\beta$ in Ref.[23], therefore the quark masses and mixing angles may receive large radiative corrections, and the successful predictions in Ref.[23] could be destroyed at low energy.

We would like to stress that the origin of all the above differences is due to the different flavor symmetry (discrete Z_9 instead of continuous $U(1)_{FN}$), different charge assignments and different flavor symmetry breaking patterns.

As most flavor models, there are a large number of operators with dimensionless order one coefficients in front of them, However, experimental tests of this model is not impossible[38]. Since quarks and leptons have their superpartners in SUSY, the flavor symmetry would affect mass matrices of squarks and sleptons as well, and specific pattern of sfermion masses would be predicted, which could be tested by measuring squark and slepton masses in future experiments. Moreover, the squarks and sleptons mass matrices are severely constrained by the experiments of flavor changing neutral current (FCNC) processes, and the off-diagonal elements of sfermion mass matrices must be suppressed in the super-CKM basis. Hence searching for FCNC and CP violating phenomena such as lepton flavor violation $\mu \to e\gamma$ and $\mu - e$ conversion in atom, electric dipole moments of the electron and neutron, and $B-\bar{B}$ mixing etc are also possible tests of the model. Moreover, we should check whether there is some accidental continuous symmetry in the scalar potential, which could affect the above FCNC processes[16, 40]. In addition the cosmological consequences of the T' flavor symmetry and its spontaneous broken deserve studying further[41].

The Yukawa superpotential consists of non-renormalizable interactions except the top quark relevant term in the model, and a lot of non-renormalizable operators are involved in

the higher order corrections. These non-renormalizable interactions may be generated from a renormalizable theory by integrating out some heavy fields[38]. Searching for the origin of these non-renormalizable interactions is a challenging and interesting question, then the free parameters of the model would be drastically reduced, and the model become more predictive.

ACKNOWLEDGEMENTS

I am grateful to Prof. Dao-Neng Gao and Mu-Lin Yan for very helpful and stimulating discussions. This work is supported by the China Postdoctoral Science foundation (20070420735).

APPENDIX A: BASIC PROPERTIES OF THE DISCRETE GROUP T'

The group T' is denoted as 24/13 in the Thomas-Wood classification[39], and it is isomorphic to the group $SL_2(F_3)$[22, 25], which consists of 2×2 unimodular matrices whose elements are added and multiplied as integers modulo 3. T' is the double cover of A_4 which is the even permutation of 4 objects, and the order of T' is 24. Geometrically, T' is proper rotations leaving a regular tetrahedron invariant in the SU(2) space. T' can be generated by two generators S and T with the multiplication rules[35, 39]

$$S^4 = T^3 = 1, \quad TS^2 = S^2T, \quad ST^{-1}S = TST \tag{A1}$$

The 24 elements can be written in the form $T^l S^m T^n$, where $l = 0, \pm1$, and if $m = 0$ or 2 then $n = 0$, while if $m = \pm1$ then $n = 0, \pm1$.

The character table, the explicit matrix representations and the Clebsch-Gordan coefficients of T' have already been calculated[35], which are reformulated in Ref.[23]. T' has seven inequivalent irreducible representations: three singlet representations $\mathbf{1}^0$ and $\mathbf{1}^{\pm}$, three doublet representations $\mathbf{2}^0$ and $\mathbf{2}^{\pm}$, and one triplet representation $\mathbf{3}$. The triality superscript can describe the multiplication rules of these representations concisely. We identify $\pm$ as ±1, trialities add under addition modulo three, and the multiplication rules are as follows

$$\mathbf{1}^i \otimes \mathbf{1}^j = \mathbf{1}^{i+j}\ , \quad \mathbf{1}^i \otimes \mathbf{2}^j = \mathbf{2}^j \otimes \mathbf{1}^i = \mathbf{2}^{i+j} \quad (\text{with } i, j = 0, \pm1)$$
$$\mathbf{1}^i \otimes \mathbf{3} = \mathbf{3} \otimes \mathbf{1}^i = \mathbf{3}\ , \quad \mathbf{2}^i \otimes \mathbf{2}^j = \mathbf{3} \oplus \mathbf{1}^{i+j}$$

$$\mathbf{2}^i \otimes \mathbf{3} = \mathbf{3} \otimes \mathbf{2}^i = \mathbf{2}^0 \oplus \mathbf{2}^+ \oplus \mathbf{2}^- \ , \mathbf{3} \otimes \mathbf{3} = \mathbf{3}_S \oplus \mathbf{3}_A \oplus \mathbf{1}^0 \oplus \mathbf{1}^+ \oplus \mathbf{1}^- \tag{A2}$$

where the triality notations are related the usually used notations $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{2}$, $\mathbf{2}'$ and $\mathbf{2}''$ [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]by the relations $\mathbf{1}^0 \equiv \mathbf{1}$, $\mathbf{1}^+ \equiv \mathbf{1}'$, $\mathbf{1}^- \equiv \mathbf{1}''$ and similarly for the doublet representations. The representations $\mathbf{1}'$ and $\mathbf{1}''$ are complex conjugated to each other, and the same for the $\mathbf{2}'$ and $\mathbf{2}''$ representations. Since T' is generated by the elements S and T, we only need explicit matrix representations of both S and T as follows

$$\begin{aligned}
&S(\mathbf{1}^0) = S(\mathbf{1}^+) = S(\mathbf{1}^-) = 1 \\
&T(\mathbf{1}^0) = 1 \ , \quad T(\mathbf{1}^+) = \omega \ , \quad T(\mathbf{1}^-) = \omega^2 \\
&S(\mathbf{2}^0) = S(\mathbf{2}^+) = S(\mathbf{2}^-) = N_1 \\
&T(\mathbf{2}^0) = \omega N_2 \ , \quad T(\mathbf{2}^+) = \omega^2 N_2 \ , \quad T(\mathbf{2}^-) = N_2 \\
&S(\mathbf{3}) = \frac{1}{3}\begin{pmatrix} -1 & 2\,\omega & 2\,\omega^2 \\ 2\,\omega^2 & -1 & 2\,\omega \\ 2\,\omega & 2\,\omega^2 & -1 \end{pmatrix} \ , \quad T(\mathbf{3}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}
\end{aligned} \tag{A3}$$

where $\omega = e^{i2\pi/3}$, and the matrices N_1 and N_2 are defined as

$$N_1 = \frac{-1}{\sqrt{3}}\begin{pmatrix} i & \sqrt{2}\,e^{i\pi/12} \\ -\sqrt{2}\,e^{-i\pi/12} & -i \end{pmatrix} \ , \quad N_2 = \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix} \tag{A4}$$

APPENDIX B: HIGHER ORDER CORRECTIONS TO THE VACUUM ALIGNMENT

We will discuss how the vacuum alignment achieved at the leading order is modified by the inclusion of higher dimensional operators, then the superpotential w_v is modified into

$$w_v = w_v^{LO} + w_v^{NL} \tag{B1}$$

where w_v^{LO} is the leading order contributions

$$\begin{aligned}
w_v^{LO} &= g_1(\varphi_T^R\phi\phi) + g_2(\varphi_T^R\varphi_T)\Delta + g_3(\phi^R\phi)\chi + g_4(\varphi_T\phi^R\phi) + g_5\chi^R\chi^2 + g_6\chi^R\theta'\theta'' \\
&+g_7\chi^R(\varphi_T\varphi_T) + g_8\theta''^R\theta''^2 + g_9\theta''^R\theta'\chi + g_{10}\theta''^R(\varphi_T\varphi_T)' + M_\Delta\Delta^R\Delta + g_{11}\Delta^R\chi^2 \\
&+g_{12}\Delta^R\theta'\theta'' + g_{13}\Delta^R(\varphi_T\varphi_T) + \bar{M}_\Delta\bar{\Delta}^R\bar{\Delta} + g_{14}\bar{\Delta}^R\Delta^2 + g_{15}(\varphi_S^R\varphi_S\varphi_S) + g_{16}(\varphi_S^R\varphi_S)\tilde{\xi} \\
&+g_{17}\xi^R(\varphi_S\varphi_S) + g_{18}\xi^R\xi^2 + g_{19}\xi^R\xi\tilde{\xi} + g_{20}\xi^R\tilde{\xi}^2
\end{aligned} \tag{B2}$$

The above leading order superpotential gives rise to the following vacuum configuration

$$\begin{aligned}&\langle\varphi_T\rangle=(v_T,0,0),\quad\langle\varphi_S\rangle=(v_S,v_S,v_S),\quad\langle\phi\rangle=(v_1,0),\\&\langle\xi\rangle=u_\xi,\quad\langle\tilde{\xi}\rangle=0,\quad\langle\theta'\rangle=u'_\theta,\quad\langle\theta''\rangle=u''_\theta,\\&\langle\Delta\rangle=u_\Delta,\quad\langle\bar{\Delta}\rangle=\bar{u}_\Delta,\quad\langle\chi\rangle=u_\chi\end{aligned}\tag{B3}$$

The effect of the next to leading order superpotential w_v^{NL} on the above SUSY conserving vacuum configuration is just a shift in the VEVs of the scalar fields, therefore the vacuum configuration is modified into

$$\begin{aligned}&\langle\varphi_T\rangle=(v_T+\delta v_{T1},\delta v_{T2},\delta v_{T3}),\quad\langle\varphi_S\rangle=(v_S+\delta v_{S1},v_S+\delta v_{S2},v_S+\delta v_{S3}),\\&\langle\phi\rangle=(v_1+\delta v_1,\delta v_2),\quad\langle\xi\rangle=u_\xi,\quad\langle\tilde{\xi}\rangle=\delta\tilde{u}_\xi,\quad\langle\theta'\rangle=u'_\theta+\delta u'_\theta,\\&\langle\theta''\rangle=u''_\theta+\delta u''_\theta,\quad\langle\Delta\rangle=u_\Delta+\delta u_\Delta,\quad\langle\bar{\Delta}\rangle=\bar{u}_\Delta+\delta\bar{u}_\Delta,\quad\langle\chi\rangle=u_\chi\end{aligned}\tag{B4}$$

and w_v^{NL} is given by

$$w_v^{NL}\;=\;\frac{1}{\Lambda}\sum_{i=1}^{14}t_i\mathcal{O}_i^T+\frac{1}{\Lambda^2}(f_1\mathcal{O}_1^\phi+\sum_{i=1}^{8}k_i\mathcal{O}_i^\chi+\sum_{i=1}^{4}c_i\mathcal{O}_i^\theta+\sum_{i=1}^{8}d_i\mathcal{O}_i^\Delta)+\frac{1}{\Lambda}\sum_{i=1}^{4}\bar{d}_i\mathcal{O}_i^{\bar{\Delta}}$$

where $\mathcal{O}_i^T$, $\mathcal{O}_1^\phi$ etc are operators linear in the driving fields φ_T^R and ϕ^R et al., which are consistent with the symmetry of the model, and each operator comprises 4 or 5 superfields. Since the next to leading order operators linear in φ_S^R and ξ^R are of order $1/\Lambda^3$, therefore the shifts $\delta v_{Si}(i=1,2,3)$ and $\delta\tilde{u}_\xi$ are suppressed by $1/\Lambda^3$, and these operators are omitted in the w_v^{NL} above. The operators $\mathcal{O}_i^T(i=1-14)$ and $\mathcal{O}_1^\phi$ are given by

$$\begin{aligned}&\mathcal{O}_1^T=(\varphi_T^R\varphi_T)(\varphi_T\varphi_T), &&\mathcal{O}_2^T=(\varphi_T^R\varphi_T)'(\varphi_T\varphi_T)'',\\&\mathcal{O}_3^T=(\varphi_T^R\varphi_T)''(\varphi_T\varphi_T)', &&\mathcal{O}_4^T=((\varphi_T^R\varphi_T)_{\mathbf{3}_S}(\varphi_T\varphi_T)_{\mathbf{3}_S}),\\&\mathcal{O}_5^T=((\varphi_T^R\varphi_T)_{\mathbf{3}_A}(\varphi_T\varphi_T)_{\mathbf{3}_S}), &&\mathcal{O}_6^T=(\varphi_T^R\varphi_T\varphi_T)\chi,\\&\mathcal{O}_7^T=(\varphi_T^R\varphi_T\varphi_T)''\theta', &&\mathcal{O}_8^T=(\varphi_T^R\varphi_T\varphi_T)'\theta'',\\&\mathcal{O}_9^T=(\varphi_T^R\varphi_T)\chi^2, &&\mathcal{O}_{10}^T=(\varphi_T^R\varphi_T)\theta'\theta'',\\&\mathcal{O}_{11}^T=(\varphi_T^R\varphi_T)''\chi\theta', &&\mathcal{O}_{12}^T=(\varphi_T^R\varphi_T)''\theta''^2,\\&\mathcal{O}_{13}^T=(\varphi_T^R\varphi_T)'\chi\theta'', &&\mathcal{O}_{14}^T=(\varphi_T^R\varphi_T)'\theta'^2\end{aligned}\tag{B5}$$

$$\mathcal{O}_1^\phi=(\phi^R\phi)\Delta\bar{\Delta}^2\tag{B6}$$

The structures $\mathcal{O}_i^{\chi}(i=1-8)$ are explicitly written as follows

$$\begin{array}{ll} \mathcal{O}_1^{\chi}=\chi^R\Delta\bar{\Delta}^2\chi, & \mathcal{O}_2^{\chi}=\chi^R\Delta(\varphi_S\varphi_S\varphi_S), \\ \mathcal{O}_3^{\chi}=\chi^R\Delta(\varphi_S\varphi_S)\xi, & \mathcal{O}_4^{\chi}=\chi^R\Delta(\varphi_S\varphi_S)\tilde{\xi}, \\ \mathcal{O}_5^{\chi}=\chi^R\Delta\xi^3, & \mathcal{O}_6^{\chi}=\chi^R\Delta\xi^2\tilde{\xi}, \\ \mathcal{O}_7^{\chi}=\chi^R\Delta\xi\tilde{\xi}^2, & \mathcal{O}_8^{\chi}=\chi^R\Delta\tilde{\xi}^3 \end{array} \tag{B7}$$

The operators involving θ''^R and Δ^R are

$$\begin{array}{ll} \mathcal{O}_1^{\theta}=\theta''^R\theta'\Delta\bar{\Delta}^2, & \mathcal{O}_2^{\theta}=\theta''^R\Delta(\varphi_S\varphi_S\varphi_S)', \\ \mathcal{O}_3^{\theta}=\theta''^R\Delta(\varphi_S\varphi_S)'\xi, & \mathcal{O}_4^{\theta}=\theta''^R\Delta(\varphi_S\varphi_S)'\tilde{\xi} \end{array} \tag{B8}$$

$$\begin{array}{ll} \mathcal{O}_1^{\Delta}=\Delta^R\Delta\bar{\Delta}^2\chi, & \mathcal{O}_2^{\Delta}=\Delta^R\Delta(\varphi_S\varphi_S\varphi_S), \\ \mathcal{O}_3^{\Delta}=\Delta^R\Delta(\varphi_S\varphi_S)\xi, & \mathcal{O}_4^{\Delta}=\Delta^R\Delta(\varphi_S\varphi_S)\tilde{\xi}, \\ \mathcal{O}_5^{\Delta}=\Delta^R\Delta\xi^3, & \mathcal{O}_6^{\Delta}=\Delta^R\Delta\xi^2\tilde{\xi}, \\ \mathcal{O}_7^{\Delta}=\Delta^R\Delta\xi\tilde{\xi}^2, & \mathcal{O}_8^{\Delta}=\Delta^R\Delta\tilde{\xi}^3 \end{array} \tag{B9}$$

and the operators $\mathcal{O}_i^{\bar{\Delta}}(i=1-4)$ are given by

$$\begin{array}{ll} \mathcal{O}_1^{\bar{\Delta}}=\bar{\Delta}^R\Delta(\varphi_T\varphi_T), & \mathcal{O}_2^{\bar{\Delta}}=\bar{\Delta}^R\Delta\chi^2, \\ \mathcal{O}_3^{\bar{\Delta}}=\bar{\Delta}^R\Delta\theta'\theta'', & \mathcal{O}_4^{\bar{\Delta}}=\bar{\Delta}^R(\varphi_T\phi\phi), \end{array} \tag{B10}$$

We perform the same minimization procedure as that in section V, again we search for the zero of the F terms associated with the driving fields, Only terms linear in the shift δv are kept, and terms of order $\delta v/\Lambda$ are neglected, then the minimization equations become

$$\begin{aligned}
&2ig_1v_1\delta v_1+g_2u_{\Delta}\delta v_{T1}+g_2v_T\delta u_{\Delta}+\frac{v_T}{\Lambda}(t_1v_T^2+\frac{4}{9}t_4v_T^2+\frac{2}{3}t_6u_{\chi}v_T+t_9u_{\chi}^2+t_{10}u'_{\theta}u''_{\theta})=0\\
&(1-i)g_1v_1\delta v_2+g_2u_{\Delta}\delta v_{T3}+\frac{v_T}{\Lambda}(\frac{2}{3}t_8u''_{\theta}v_T+t_{13}u''_{\theta}u_{\chi}+t_{14}u'^2_{\theta})=0\\
&g_2u_{\Delta}\delta v_{T2}+\frac{v_T}{\Lambda}(\frac{2}{3}t_7u'_{\theta}v_T+t_{11}u'_{\theta}u_{\chi}+t_{12}u''^2_{\theta})=0\\
&(g_3u_{\chi}+g_4v_T)\delta v_2-(1-i)g_4v_1\delta v_{T3}=0\\
&g_4v_1\delta v_{T1}-\frac{f_1}{\Lambda^2}u_{\Delta}\bar{u}_{\Delta}^2v_1=0\\
&g_6u'_{\theta}\delta u''_{\theta}+g_6u''_{\theta}\delta u'_{\theta}+2g_7v_T\delta v_{T1}+\frac{u_{\Delta}}{\Lambda^2}(k_1\bar{u}_{\Delta}^2u_{\chi}+3k_3u_{\xi}v_S^2+k_5u_{\xi}^3)=0\\
&2g_8u''_{\theta}\delta u''_{\theta}+g_9u_{\chi}\delta u'_{\theta}+2g_{10}v_T\delta v_{T2}+\frac{u_{\Delta}}{\Lambda^2}(c_1u'_{\theta}\bar{u}_{\Delta}^2+3c_3u_{\xi}v_S^2)=0\\
&M_{\Delta}\delta u_{\Delta}+g_{12}u'_{\theta}\delta u''_{\theta}+g_{12}u''_{\theta}\delta u'_{\theta}+2g_{13}v_T\delta v_{T1}+\frac{u_{\Delta}}{\Lambda^2}(d_1\bar{u}_{\Delta}^2u_{\chi}+3d_3u_{\xi}v_S^2+d_5u_{\xi}^3)=0\\
&\bar{M}_{\Delta}\delta\bar{u}_{\Delta}+2g_{14}u_{\Delta}\delta u_{\Delta}+\frac{1}{\Lambda}(\bar{d}_1u_{\Delta}v_T^2+\bar{d}_2u_{\Delta}u_{\chi}^2+\bar{d}_3u_{\Delta}u'_{\theta}u''_{\theta}+i\bar{d}_4v_1^2v_T)=0
\end{aligned} \tag{B11}$$

Solving the above linear equations, then the shifts of the VEVs are

$$
\begin{aligned}
\delta v_{T1} &= \frac{f_1}{g_4}\frac{u_\Delta \bar{u}_\Delta^2}{\Lambda^2} \\
\delta v_{T2} &= -\frac{v_T}{g_2 u_\Delta \Lambda}(\frac{2}{3}t_7 u'_\theta v_T + t_{11}u'_\theta u_\chi + t_{12}u''^2_\theta) \\
\delta v_{T3} &= -\frac{v_T}{2g_2 u_\Delta \Lambda}(\frac{2}{3}t_8 u''_\theta v_T + t_{13}u''_\theta u_\chi + t_{14}u'^2_\theta) \\
\delta v_1 &= \frac{iv_T}{2g_1 v_1 \Lambda}(t_1 v_T^2 + \frac{4}{9}t_4 v_T^2 + \frac{2}{3}t_6 u_\chi v_T + t_9 u_\chi^2 + t_{10}u'_\theta u''_\theta) + \mathcal{O}(\frac{1}{\Lambda^2}) \\
\delta v_2 &= -\frac{(1-i)v_1}{4g_2 u_\Delta \Lambda}(\frac{2}{3}t_8 u''_\theta v_T + t_{13}u''_\theta u_\chi + t_{14}u'^2_\theta) \\
\delta u'_\theta &= -\frac{2g_{10}u'_\theta v_T^2}{3g_2 g_8 u''^2_\theta u_\Delta \Lambda}(\frac{2}{3}t_7 u'_\theta v_T + t_{11}u'_\theta u_\chi + t_{12}u''^2_\theta) + \mathcal{O}(\frac{1}{\Lambda^2}) \\
\delta u''_\theta &= \frac{2g_{10}v_T^2}{3g_2 g_8 u''_\theta u_\Delta \Lambda}(\frac{2}{3}t_7 u'_\theta v_T + t_{11}u'_\theta u_\chi + t_{12}u''^2_\theta) + \mathcal{O}(\frac{1}{\Lambda^2}) \\
\delta u_\Delta &= \frac{2(g_7 g_{12} - g_6 g_{13})f_1}{g_4 g_6}\frac{u_\Delta \bar{u}_\Delta^2 v_T}{M_\Delta \Lambda^2} + \frac{g_{12}u_\Delta}{g_6 M_\Delta \Lambda^2}(k_1 \bar{u}_\Delta^2 u_\chi + 3k_3 u_\xi v_S^2 + k_5 u_\xi^3) \\
&\quad -\frac{u_\Delta}{M_\Delta \Lambda^2}(d_1 \bar{u}_\Delta^2 u_\chi + 3d_3 u_\xi v_S^2 + d_5 u_\xi^3) \\
\delta \bar{u}_\Delta &= -\frac{1}{\bar{M}_\Delta \Lambda}(\bar{d}_1 u_\Delta v_T^2 + \bar{d}_2 u_\Delta u_\chi^2 + \bar{d}_3 u_\Delta u'_\theta u''_\theta + i d_4 v_1^2 v_T) + \mathcal{O}(\frac{1}{\Lambda^2})
\end{aligned}
\tag{B12}
$$

where the contributions of order $1/\Lambda^2$ in δv_1, $\delta u'_\theta$, $\delta u''_\theta$ and $\delta \bar{u}_\Delta$, which are not written out explicitly, are also higher order in λ relative to the leading contributions suppressed by $1/\Lambda$. Since δv_{T1} and δu_Δ are of order $1/\Lambda^2$, terms of the same order should not be omitted in the relevant minimization equations, then δv_{T1} is modified into

$$
\begin{aligned}
\delta v_{T1} &= \frac{f_1}{g_4}\frac{u_\Delta \bar{u}_\Delta^2}{\Lambda^2} - \frac{v_T}{2g_2^2 u_\Delta^2 \Lambda^2}(\frac{2}{3}t_7 u'_\theta v_T + t_{11}u'_\theta u_\chi + t_{12}u''^2_\theta)(\frac{2}{3}t_8 u''_\theta v_T + t_{13}u''_\theta u_\chi \\
&\quad + t_{14}u'^2_\theta)
\end{aligned}
\tag{B13}
$$

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